Hadron Spectroscopy:
Why and How
(Lecture 1)

National Nuclear Physics Summer School
June 2021

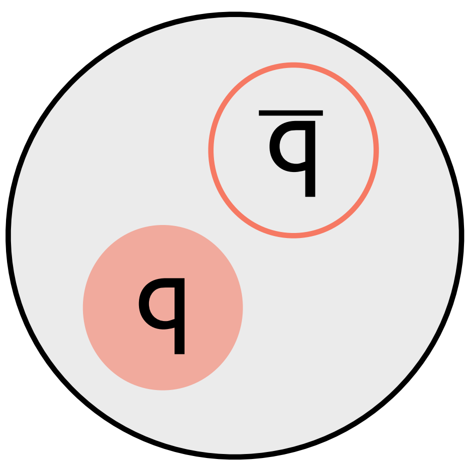
Matthew Shepherd
Indiana University

Outline

1. Hadron Spectroscopy: Why and How
    1.1. Unique features of QCD
    1.2. Why use spectroscopy as a tool to study QCD?
    1.3. How do we classify mesons?
    1.4. Introduction to experiment
2. Current Results in Hadron Spectroscopy
    2.1. Heavy Quark Spectroscopy
    2.2. Light Quark Spectroscopy
    2.3. Summary and Outlook

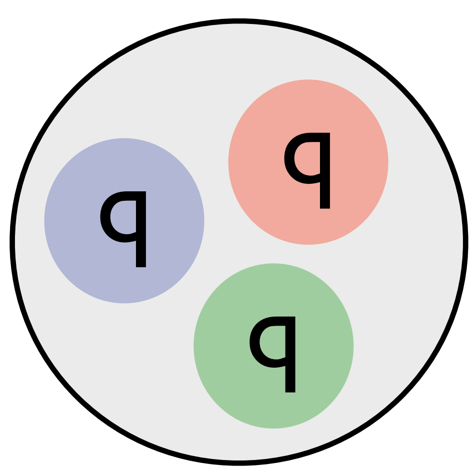
QCD in the Standard Model

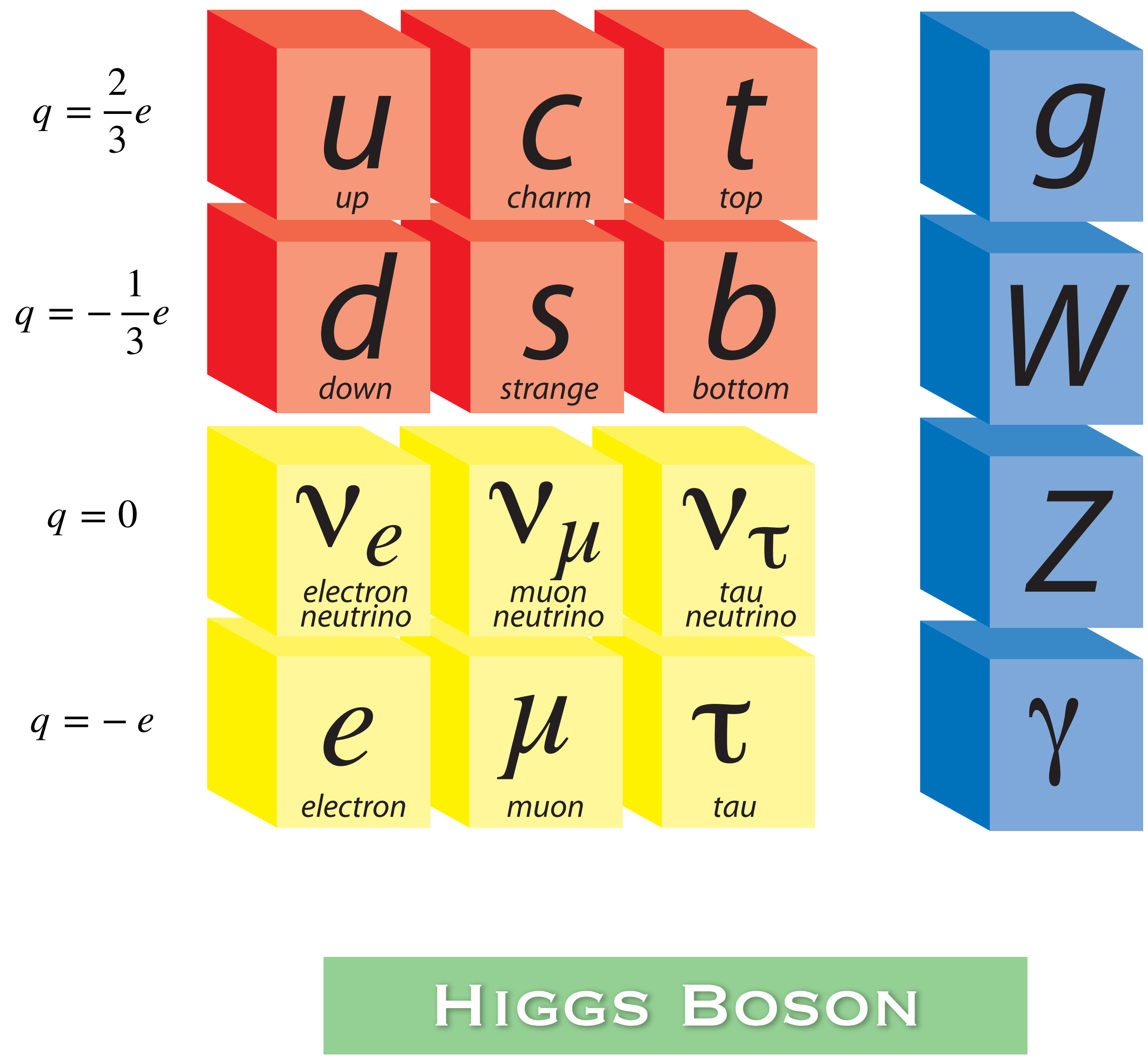
- Three quark colors
- Colorless hadrons apparently required
- Two typical arrangements: mesons and baryons



Mesons
(e.g., π, K, D)

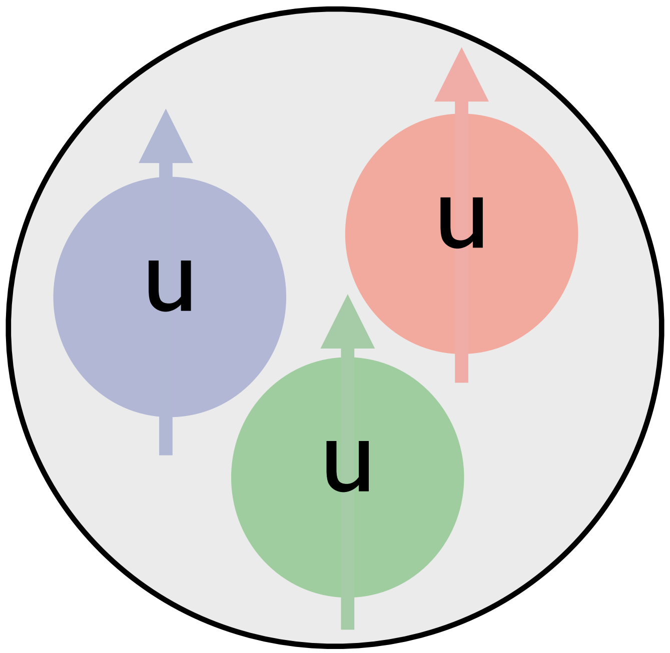
Baryons
(e.g., proton and neutron)





Evidence of Color

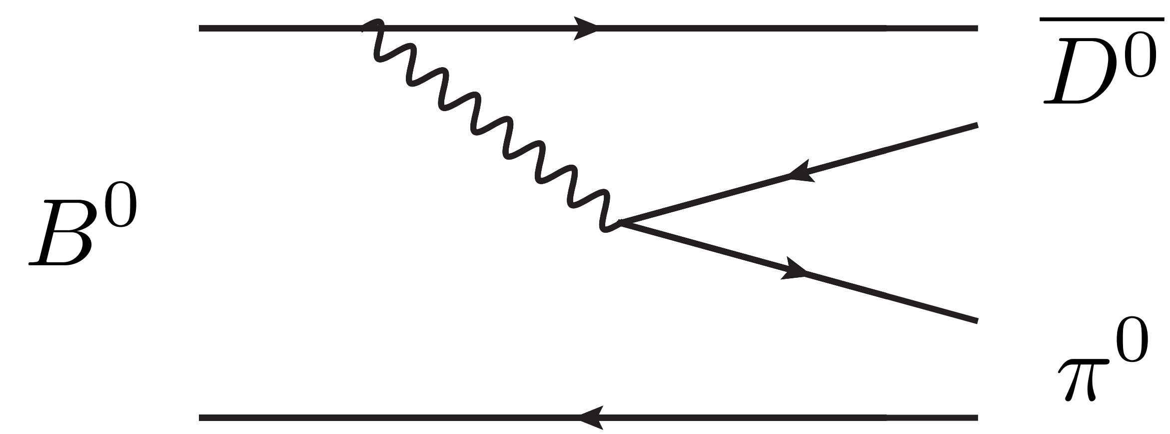
$$\Delta^{++}$$



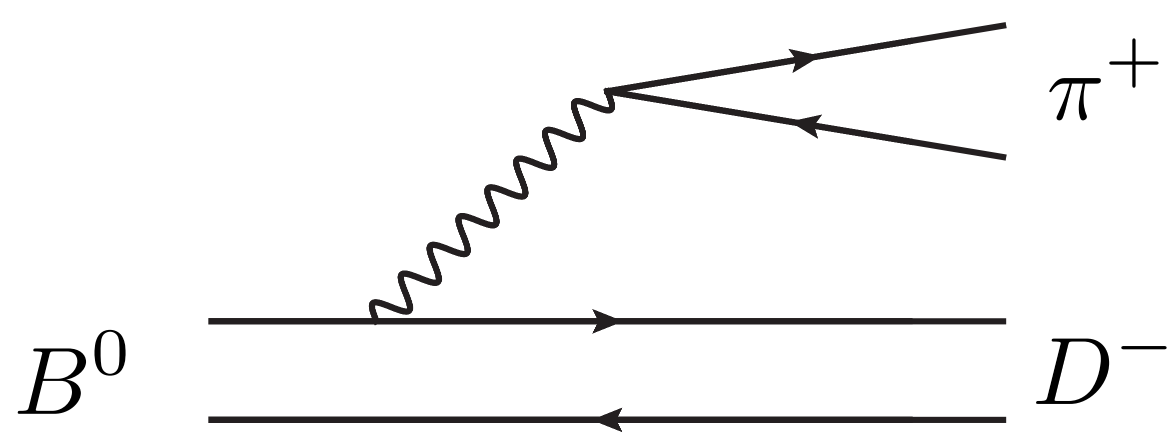
$$J = \frac{3}{2}$$

$$\frac{1}{\sqrt{6}}(rgb - rbg + brg - bgr + gbr - grb)$$

$$\mathcal{B}(B^0 \to \overline{D^0}\pi^0) = 0.26 \times 10^{-3}$$



$$\mathcal{B}(B^0 \to D^-\pi^+) = 2.7 \times 10^{-3}$$



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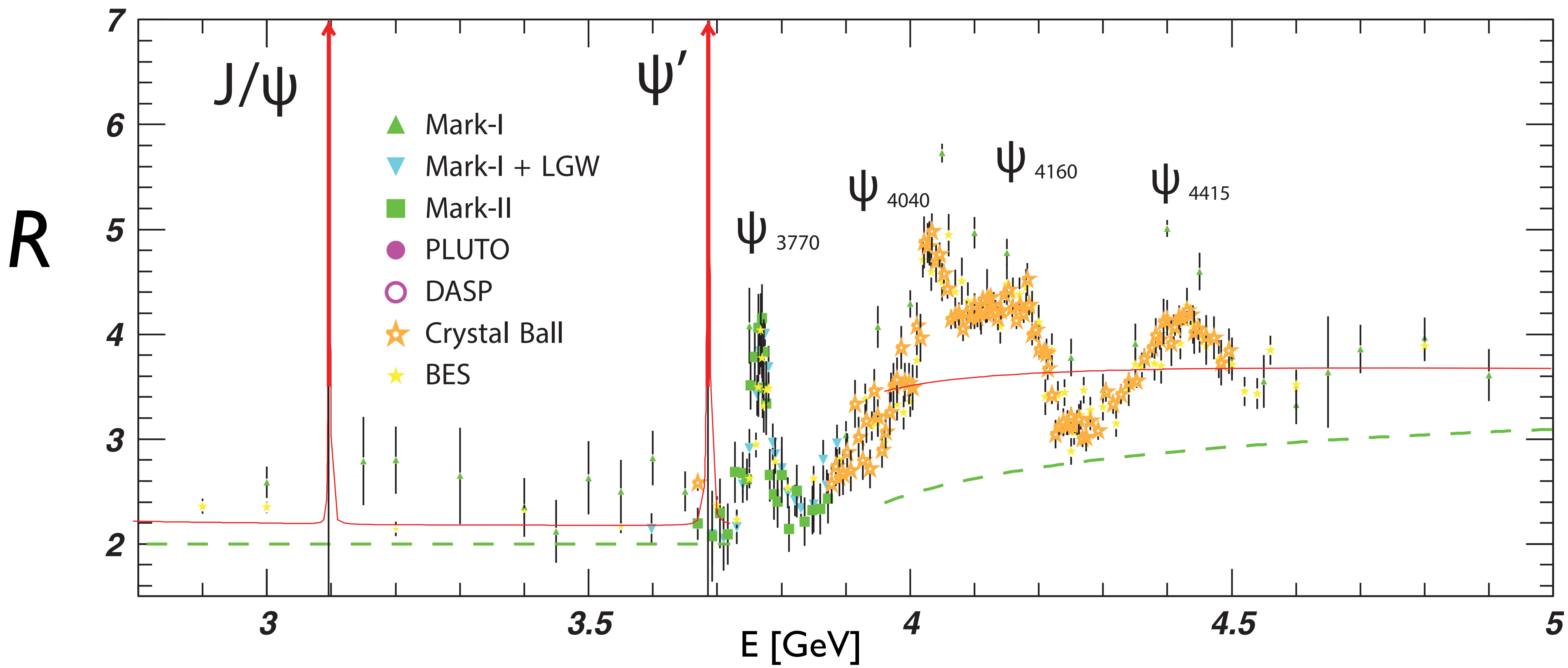
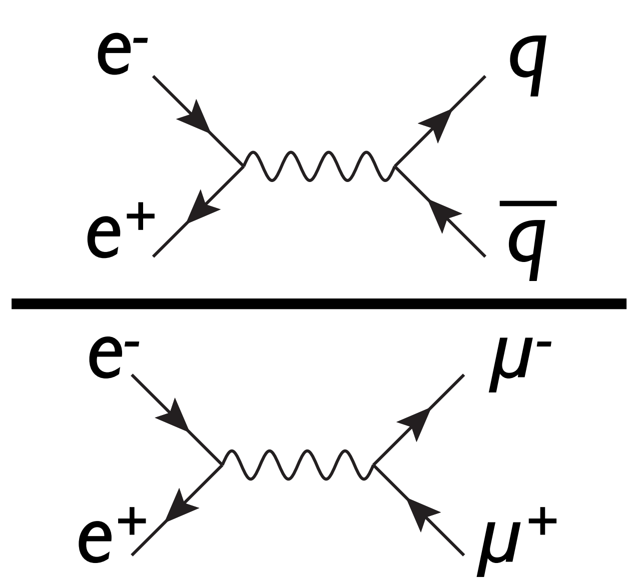
More Evidence of 3 Colors



- Probes the ratio of quark to lepton couplings in QED: Q_q^2 / Q_μ^2

$R =$ 

Homework: Compute the expected value of R below and above charm (and bottom) thresholds under the assumptions that there are 1 and 3 colors of quarks. Compare with experimental data from the PDG.

Interactions in QED

Have: freely propagating spin-1/2 particle

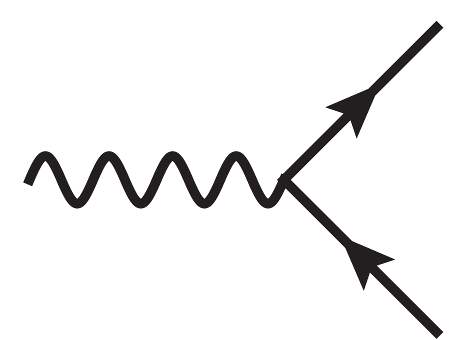
$$\mathcal{L} = i(\hbar c)\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - (mc^2)\bar{\psi}\psi$$

Want: “physics” to remain invariant under local phase transformations

$$\psi \rightarrow e^{i\theta(x)}\psi$$

Doing so requires introduction of a freely propagating massless gauge field (the photon) and the interaction of this field with spin-1/2 particles

$$-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$



$$-Q(\bar{\psi}\gamma^{\mu}\psi)A_{\mu}$$

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Interactions in QCD

Have: freely propagating spin-1/2 quark in *rgb* space

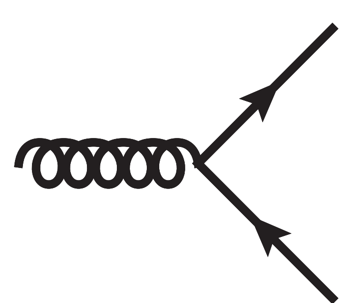
$$\mathcal{L} = i(\hbar c)\bar{\psi}\gamma^\mu\partial_\mu\psi - (mc^2)\bar{\psi}\psi$$

Want: "physics" to remain invariant under unitary color transformations

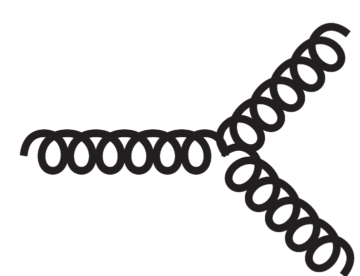
$$\begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix} \rightarrow U \begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix}$$

This requires the introduction of eight massless gauge fields (the gluons) and several interaction terms -- *note that gluons interact with each other!*

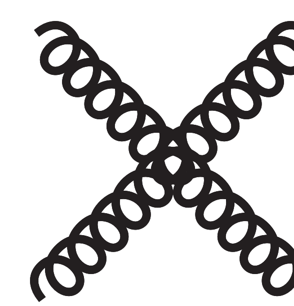
gluons



quark-gluon vertex



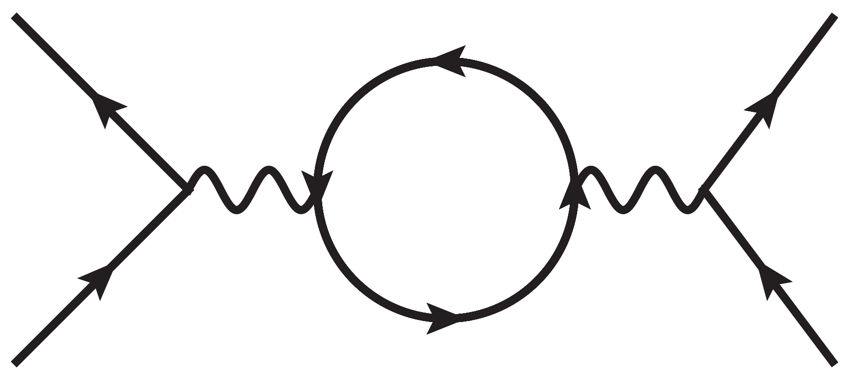
three-gluon vertex



four-gluon vertex

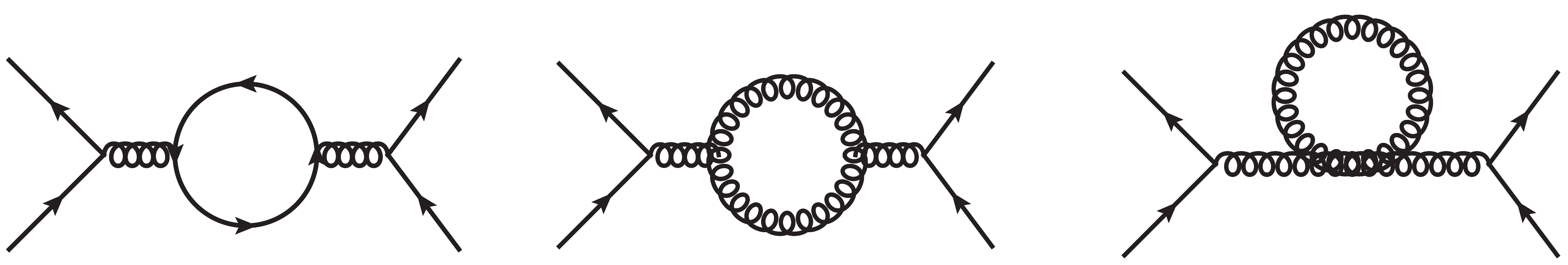
Higher Order Corrections

- In QED, vacuum polarization acts to "screen" the charges of interacting particles resulting in *weaker force at large distance*.



scale of corrections set by
$\alpha = 1/137$

- In QCD quark loops continue to screen the QCD force, but gluon loops provide an "anti-screening" effect that dominates, resulting in a *stronger force at large distances*.

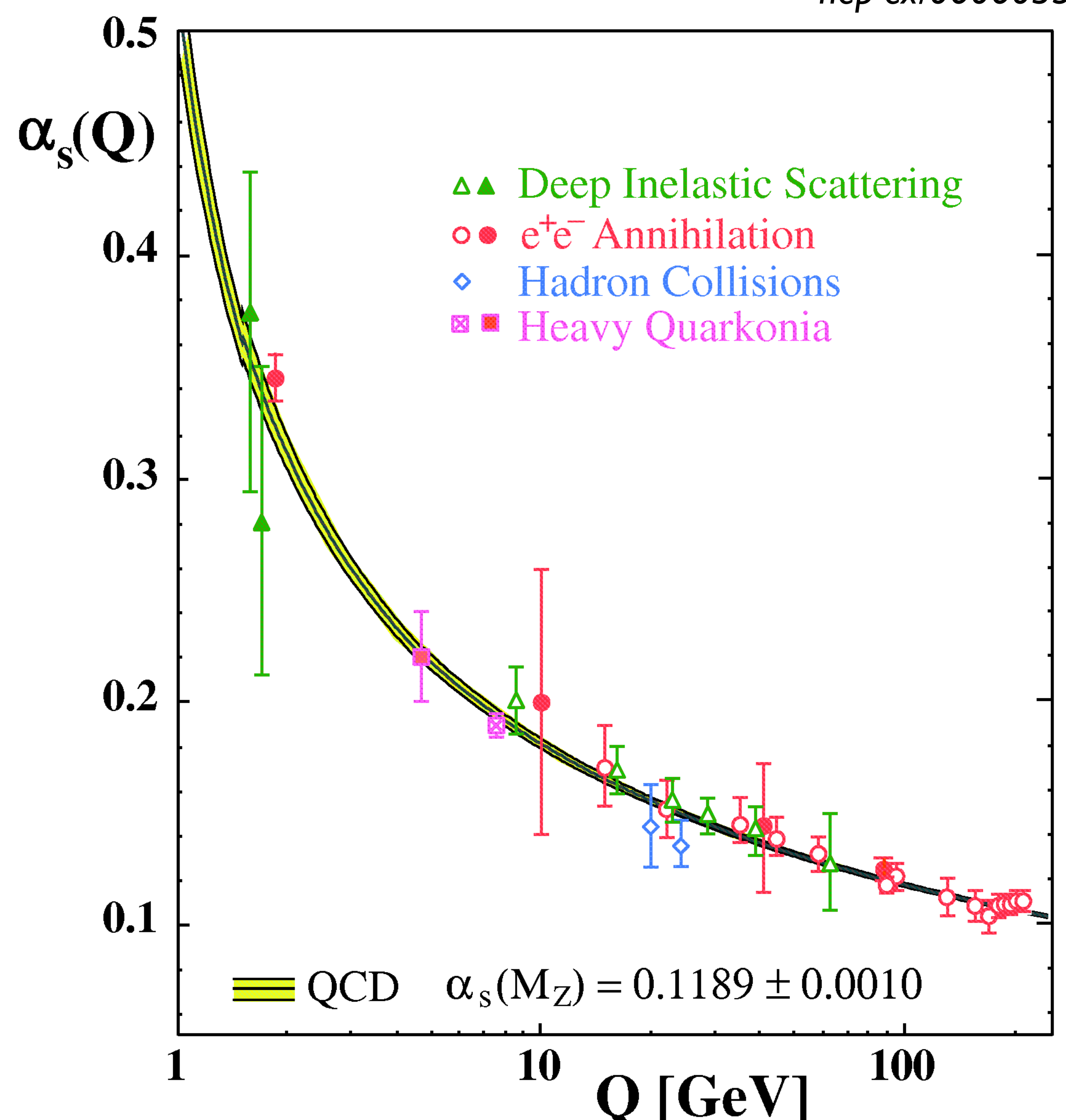


scale of QCD corrections set by $\alpha_s > 0.1$

Gluon Interactions in QCD

- QCD has interesting properties
  - gluon-gluon interactions
  - confinement
- Nonpurturbative in the interesting domain
- *Study QCD using hadrons*

S. Bethke
hep-ex/0606035



long distance

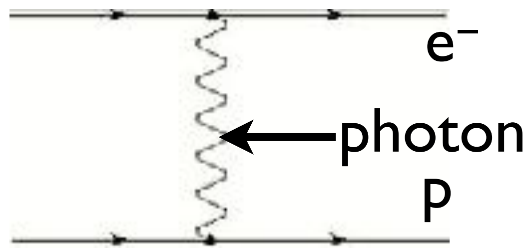
short distance

The Forces that Bind Hydrogen and Mesons

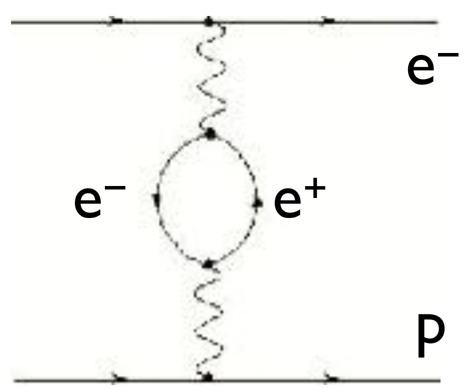
The Electromagnetic Force and Quantum Electrodynamics (QED)

(Hydrogen Atom)

Simple Term

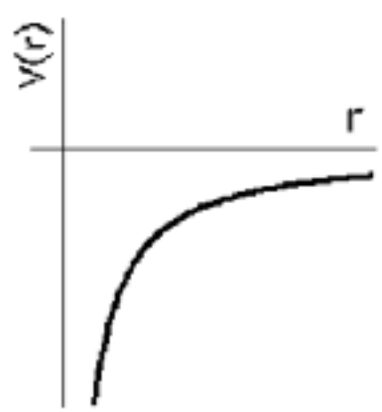


Correction



Potential

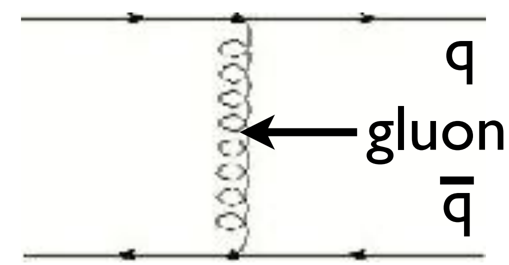
$$V(r) = -\frac{\alpha}{r}$$



IONIZATION IS POSSIBLE

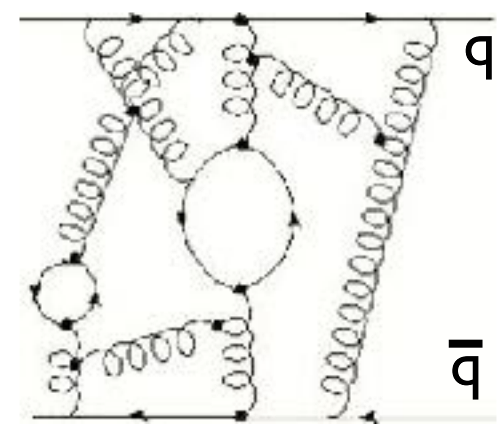
The Strong Force and Quantum Chromodynamics (QCD)

(Meson)



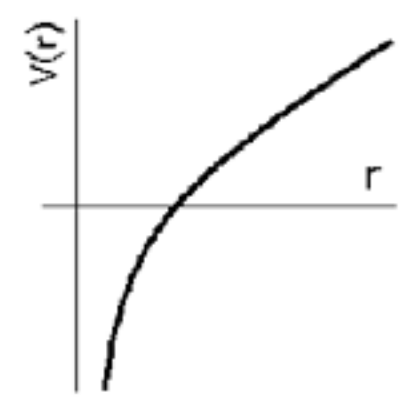
Simple Term

(small distances)



"Correction"

(large distances)



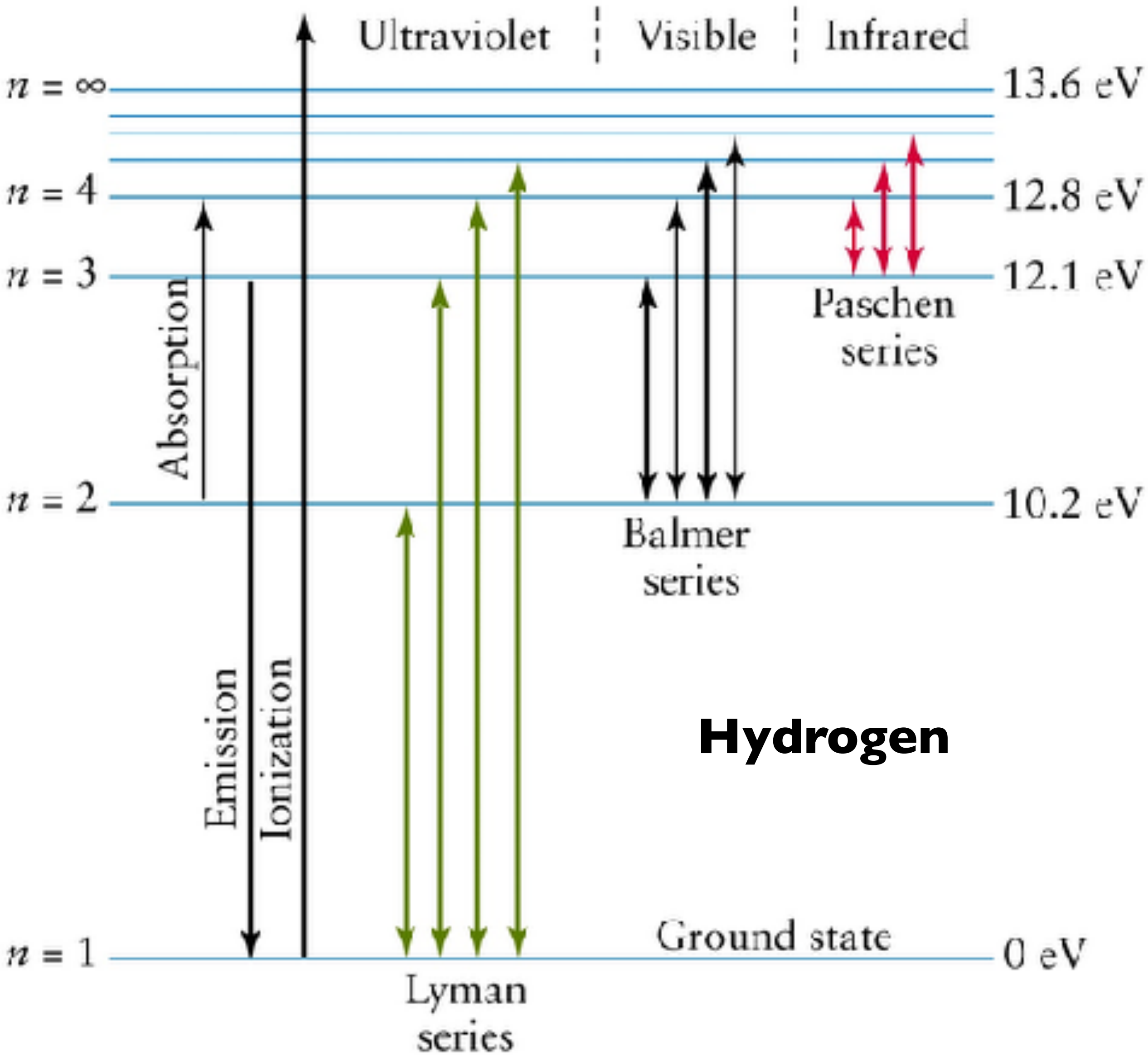
Potential (model)

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + F_0 r$$

QUARKS ARE CONFINED

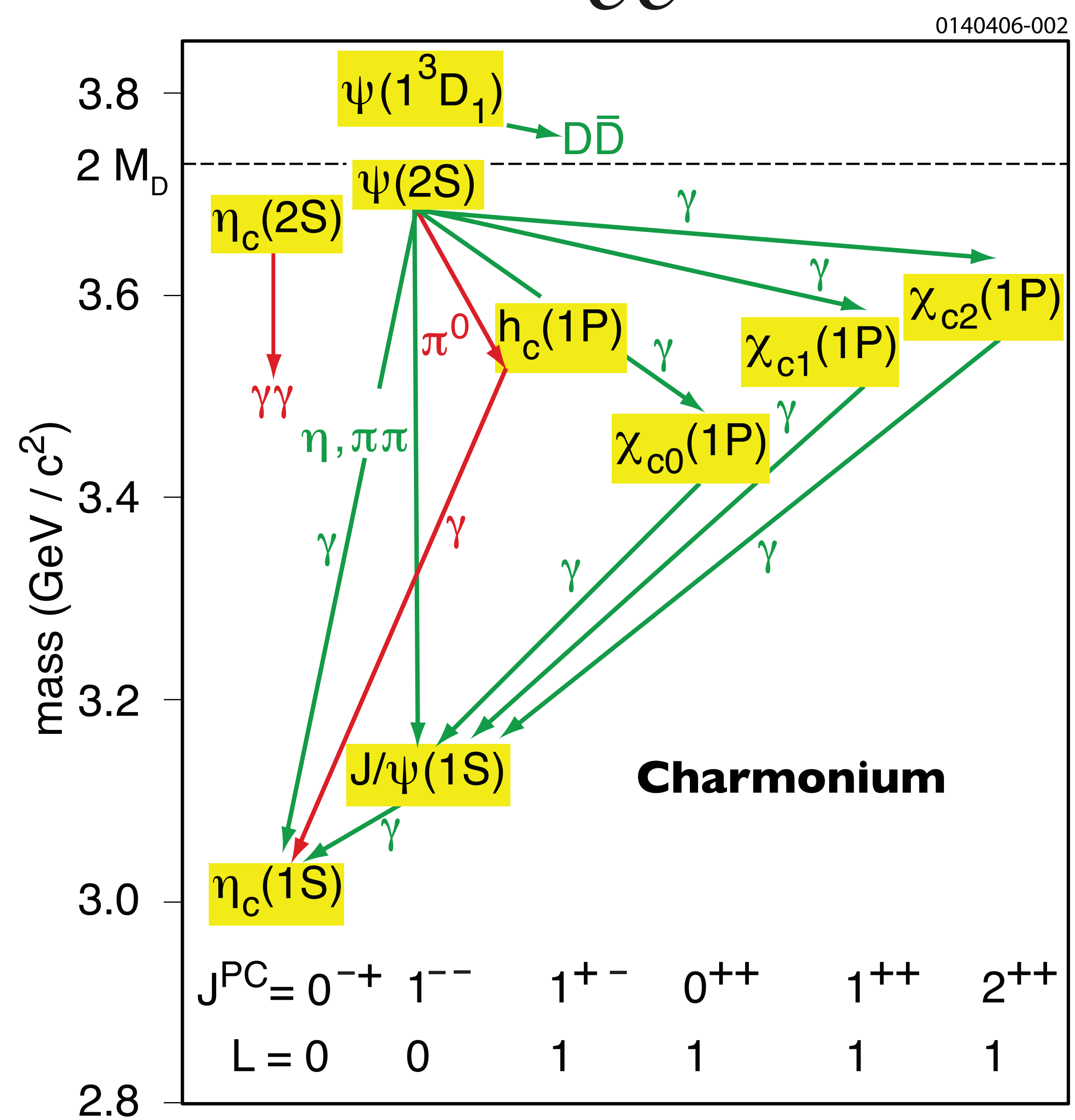
Studying Forces with Spectroscopy

Electromagnetic Force

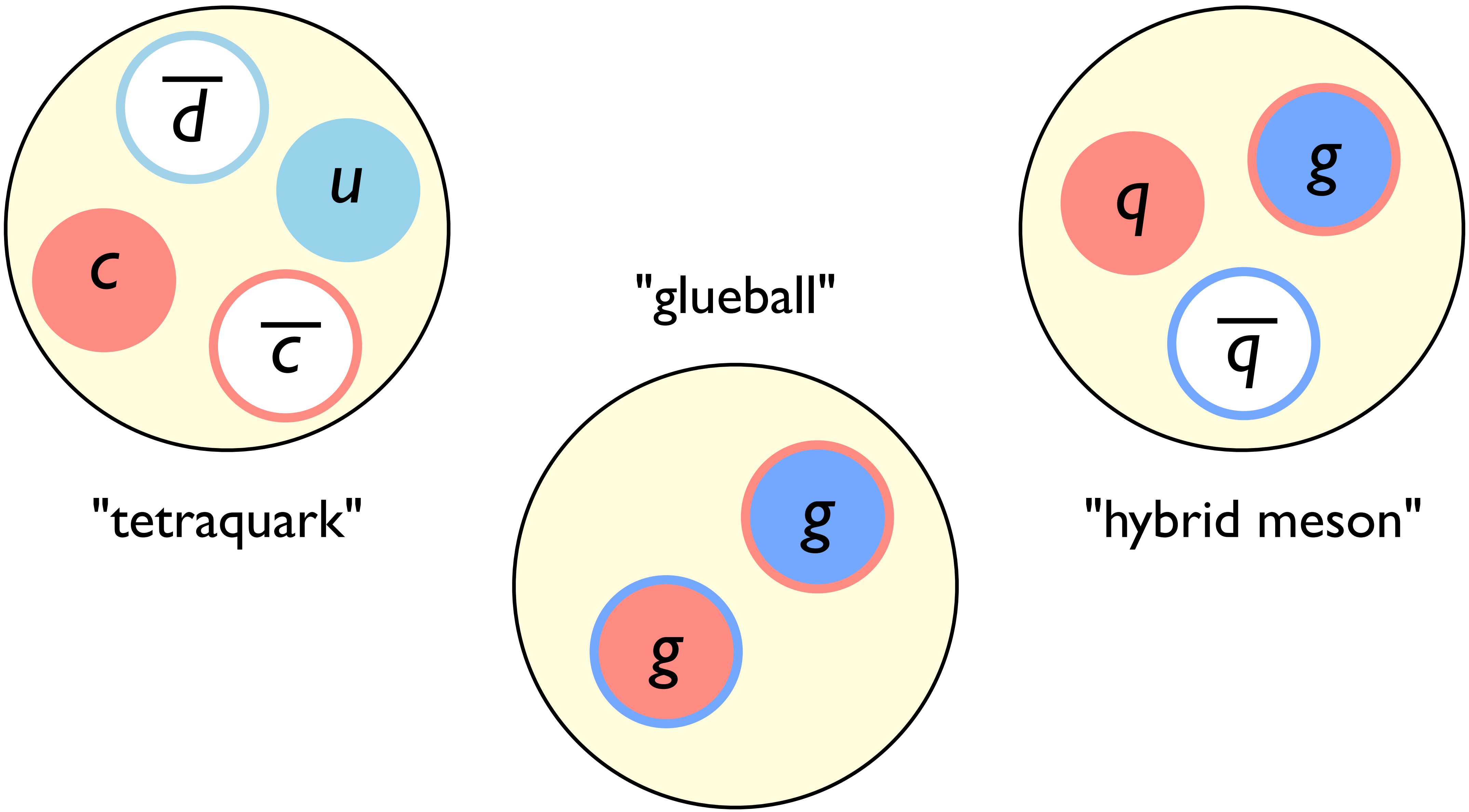


Strong Force

$c\bar{c}$



Other Configurations of Quarks and Gluons



Not forbidden by the requirement
of colorless hadrons -- do they exist in nature?

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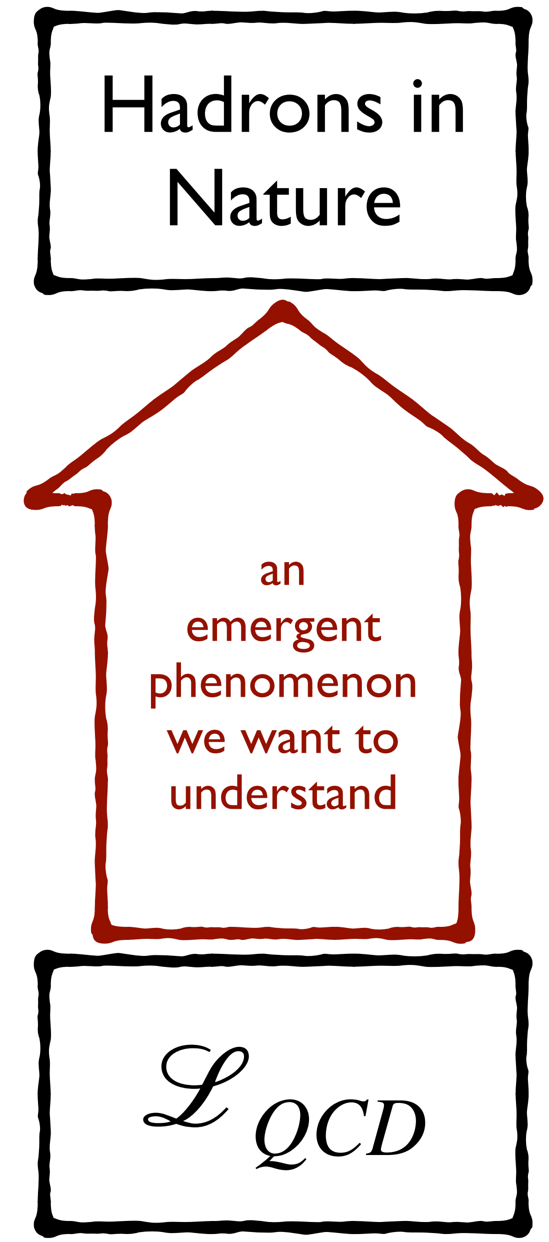
Spectroscopy and QCD

- Studying the spectrum of hadrons motivated the quark model and led to development of QCD
- QCD has interesting properties
  - confinement: force is strong at large distances
    - colorless hadrons that can be made with any number of quarks
  - gluon-gluon interactions: how do they affect the spectrum and properties of hadrons?
- *Why is the spectrum of hadrons observed in nature so simple? (...or is it?)*



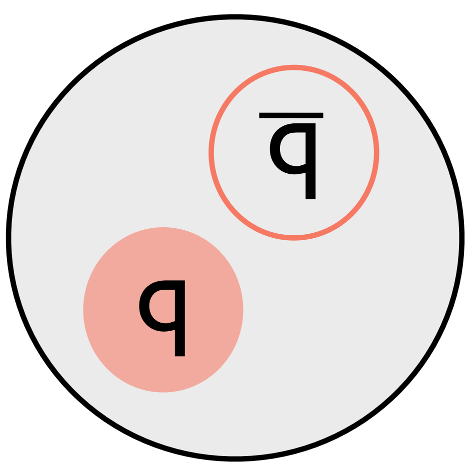
Classifying Mesons

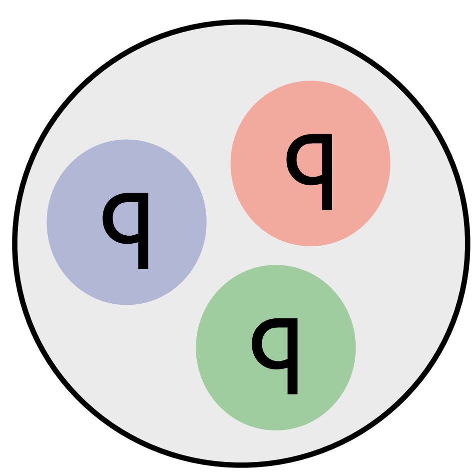
Properties of Mesons

- mass
- width or lifetime
- spin
- parity
- charge conjugation
- charge and isospin

Constituent Quark Model

- Assemble mesons from spin 1/2 constituent quarks with effective masses
    - a model: not the quark fields in the QCD Lagrangian

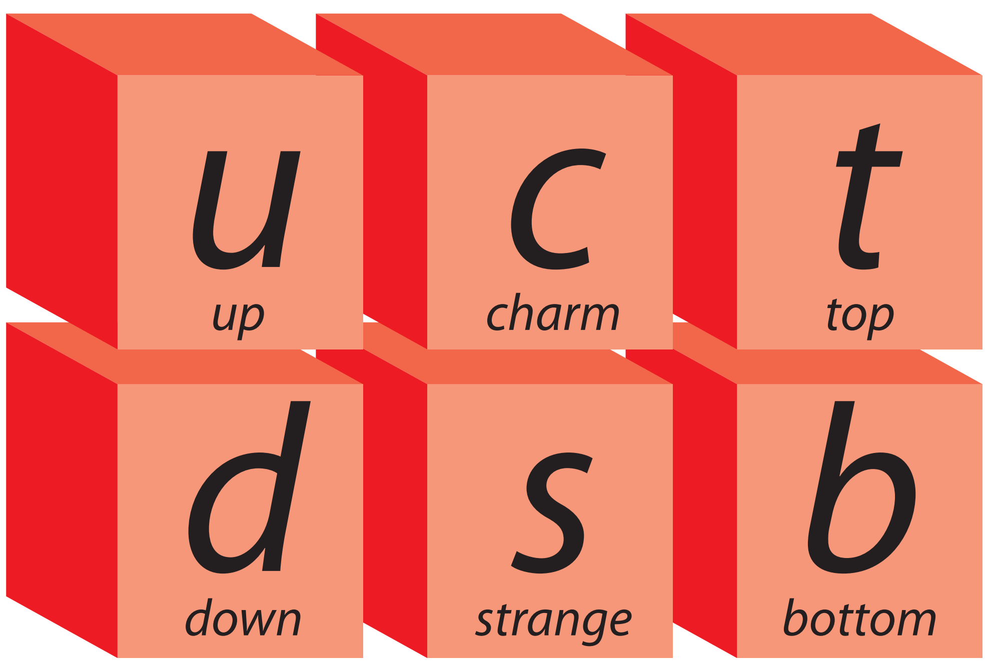




1.5 GeV

177 GeV (unstable)

few hundred MeV



$$q = \frac{2}{3}e$$

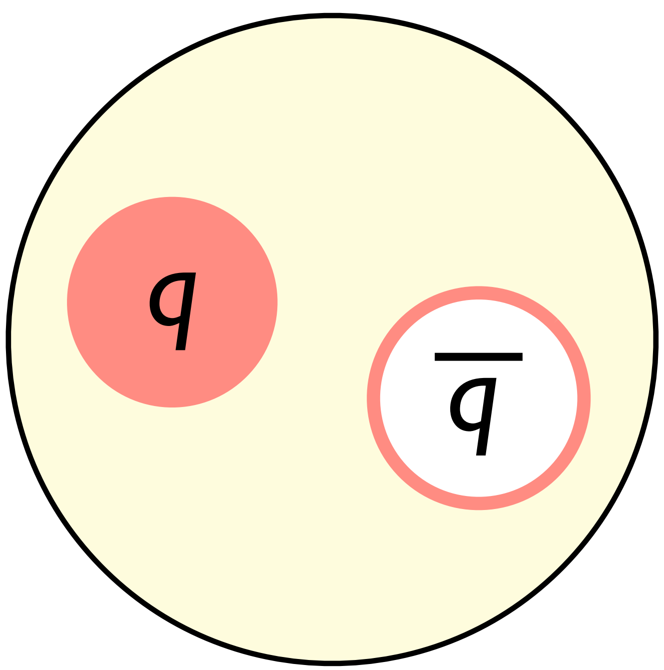
$$q = -\frac{1}{3}e$$

0.5 GeV

4.7 GeV

J^{PC}

color singlet
quark anti-quark



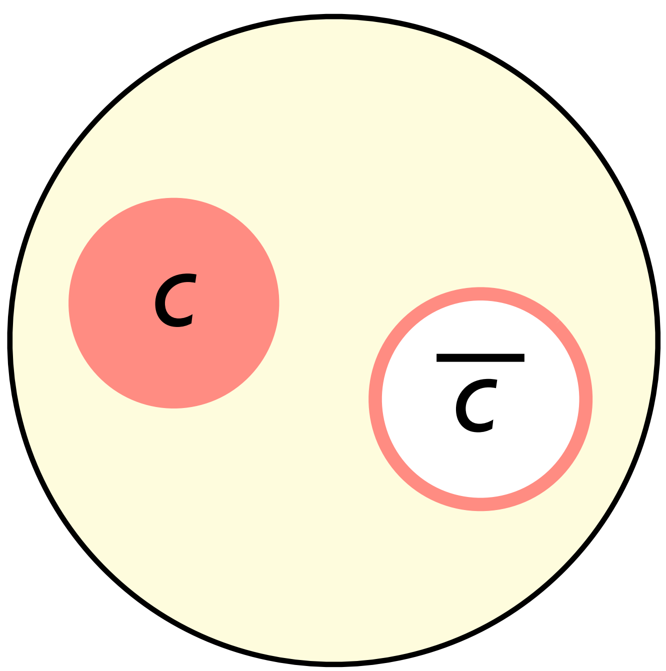
$$\vec{J} = \vec{L} + \vec{S} \quad P = (-1)^{L+1} \quad C = (-1)^{L+S}$$

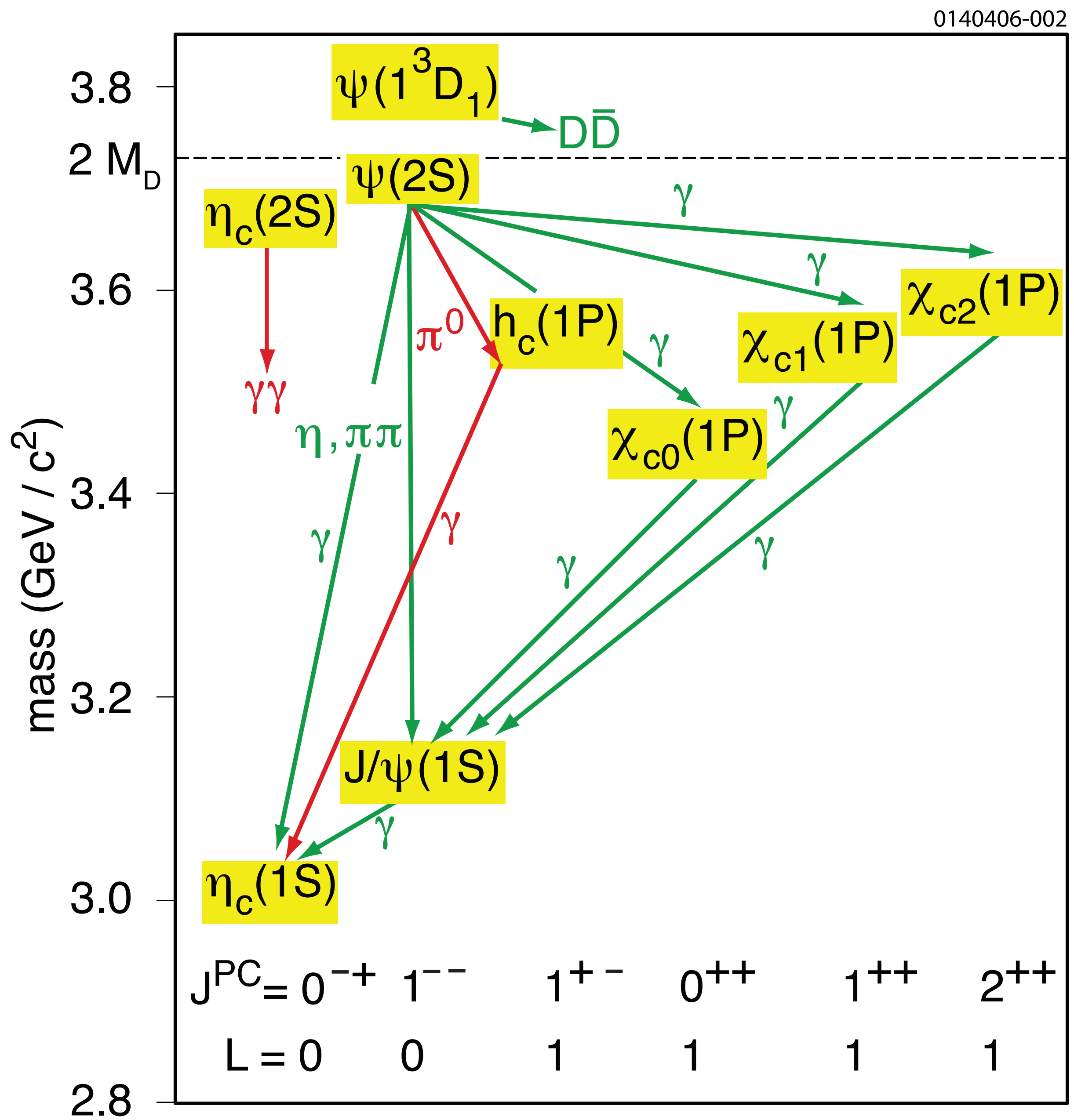
S = 0 or 1, and L = 0, 1, 2, …

- P: inverts coordinates
  - quark wave function is odd under spatial inversion for L odd: $(-1)^L$
  - intrinsic party of quark anti-quark: 1 x -1 = -1
- C: particle → anti-particle
  - neutral eigenstates
  - spatial inversion: $(-1)^L$
  - fermion → anti-fermion: -1
  - opposite spins: -1^{S+1}

Charmonium Spectrum

- All states below 2 M_D observed
- No extra states below 2 M_D
- Good agreement with potential model calculation
- Structure is a quark anti-quark each with spin-$\frac{1}{2}$ and having a mass of about 1.5 GeV





$$\vec{J} = \vec{L} + \vec{S} \quad P = (-1)^{L+1} \quad C = (-1)^{L+S}$$

Isospin

- symmetry from $m_u \approx m_d$
- in isospin space:
  u has $I = 1/2$, $I_z = 1/2$
  d has $I = 1/2$, $I_z = -1/2$
- Combining quark antiquark elements from this vector space gives four combinations (examples for 0^{-+} given)

$$|u\bar{d}\rangle \rightarrow \pi^+$$

$$\frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right) \rightarrow \pi^0$$

$$|d\bar{u}\rangle \rightarrow \pi^-$$

triplet: isovector (I = 1)

Homework: why are these almost exactly a factor of two different?

$$\mathcal{B}(\psi(2S) \rightarrow \pi^+\pi^- J/\psi) = 0.34$$

$$\mathcal{B}(\psi(2S) \rightarrow \pi^0\pi^0 J/\psi) = 0.18$$

Homework: why is the first so much bigger than the second even though there is less phase space available?

$$\mathcal{B}(\psi(2S) \rightarrow \eta J/\psi) = 0.034$$

$$\mathcal{B}(\psi(2S) \rightarrow \pi^0 J/\psi) = 0.0013$$

$$\frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle\right) \rightarrow \eta$$

singlet: isoscalar (I = 0)

G Parity

- Extension of C to isovectors (charged particles):
  - apply C
  - rotate by π in isospin space: u ↔ d
- Multiplicative
- Mostly conserved in strong interactions
- *general: G = C(-1)^I*

$$|u\bar{d}\rangle \to |\bar{u}d\rangle \to |\bar{d}u\rangle$$

isospin: 0 : G = C

$$|u\bar{u}\rangle + |d\bar{d}\rangle \to |d\bar{d}\rangle + |u\bar{u}\rangle$$

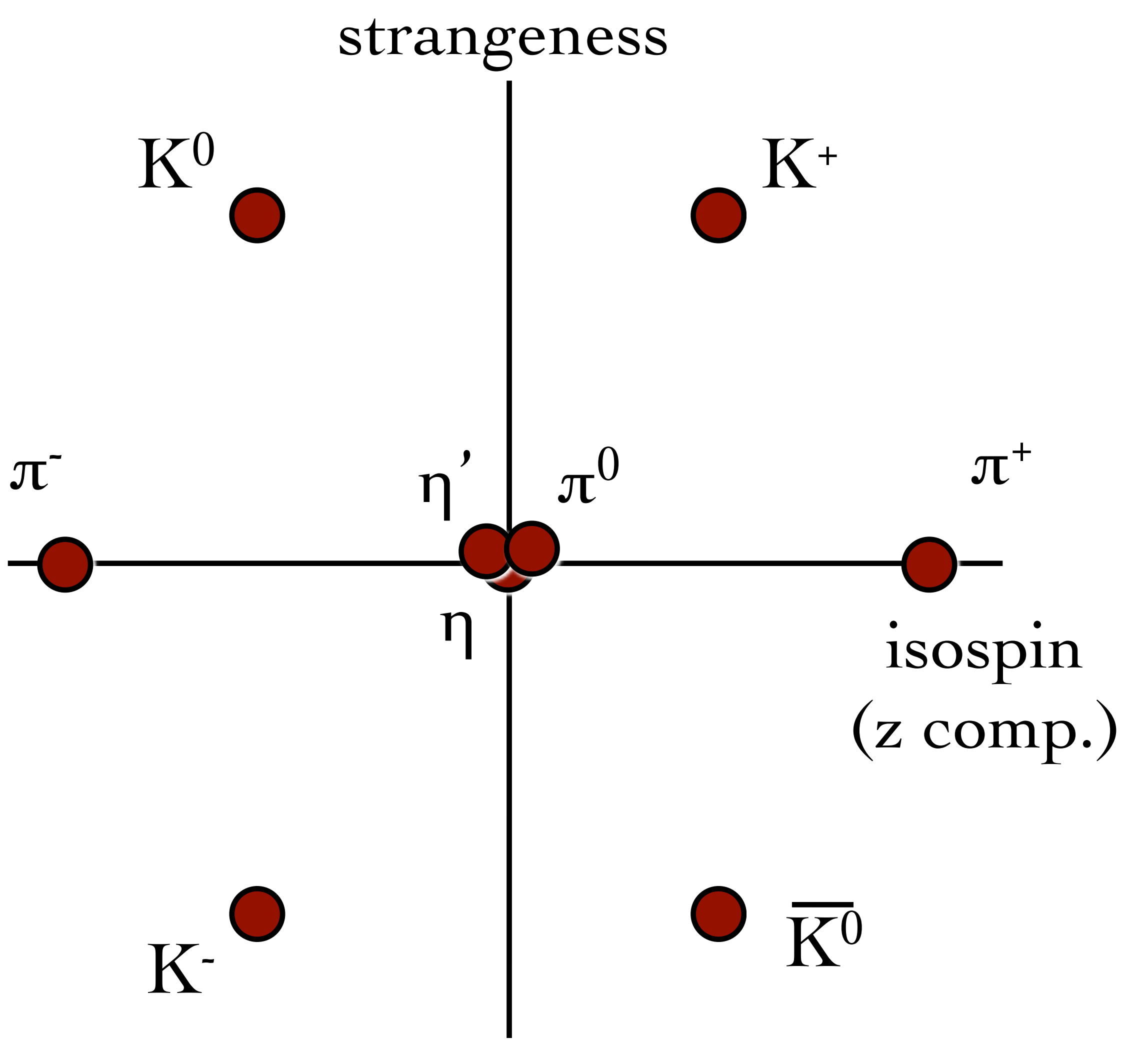
isospin: 1 : G = -C

$$|u\bar{u}\rangle - |d\bar{d}\rangle \to |d\bar{d}\rangle - |u\bar{u}\rangle$$

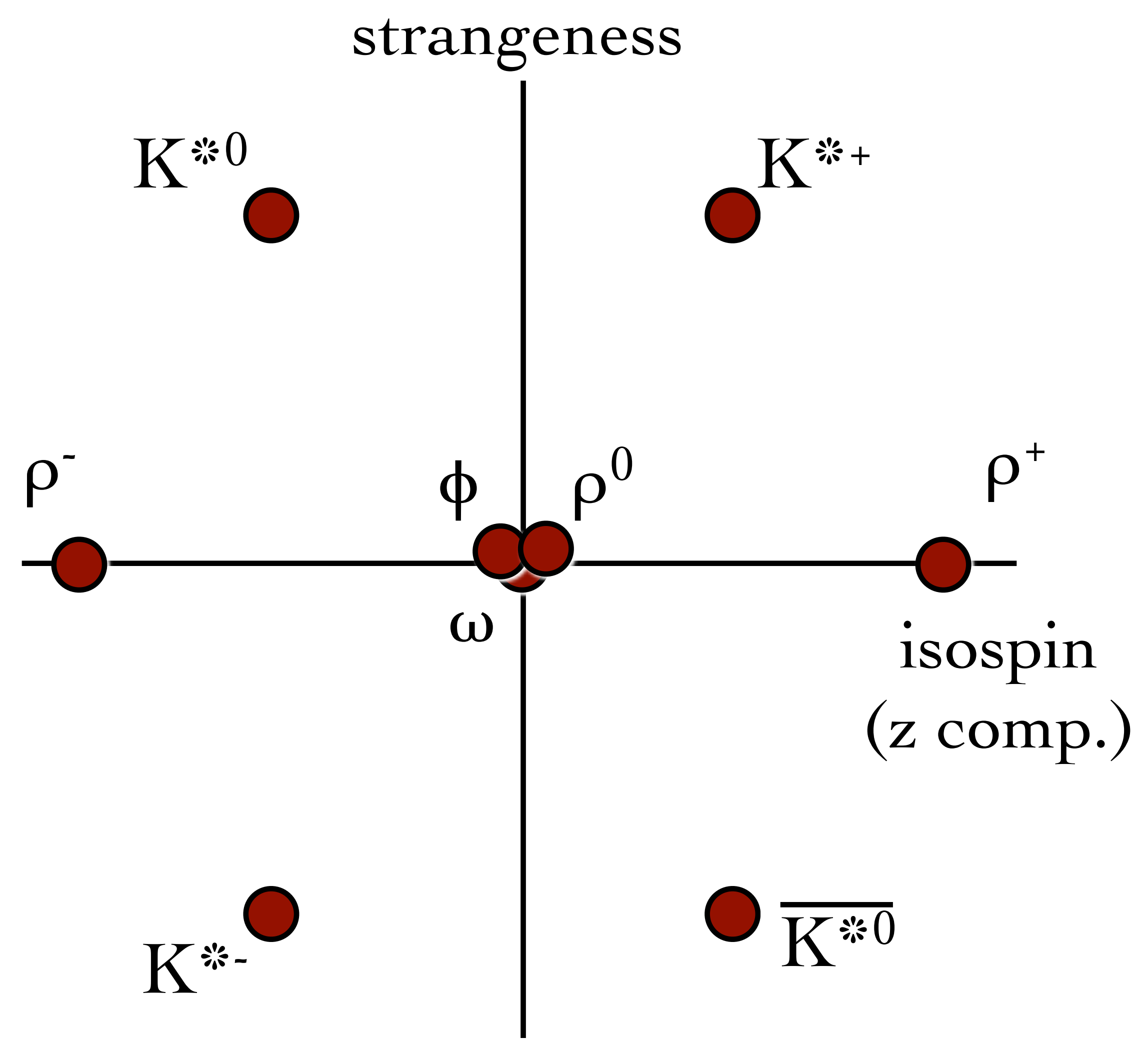
$$B(\rho^0 \to \pi^+\pi^-) \approx 1$$
$$B(\rho^0 \to \pi^+\pi^-\pi^0) = 1 \times 10^{-4}$$

$$B(\omega \to \pi^+\pi^-) = 0.015$$
$$B(\omega \to \pi^+\pi^-\pi^0) = 0.89$$

Light Quark Nonets



0^- nonet
neutrals: C=+



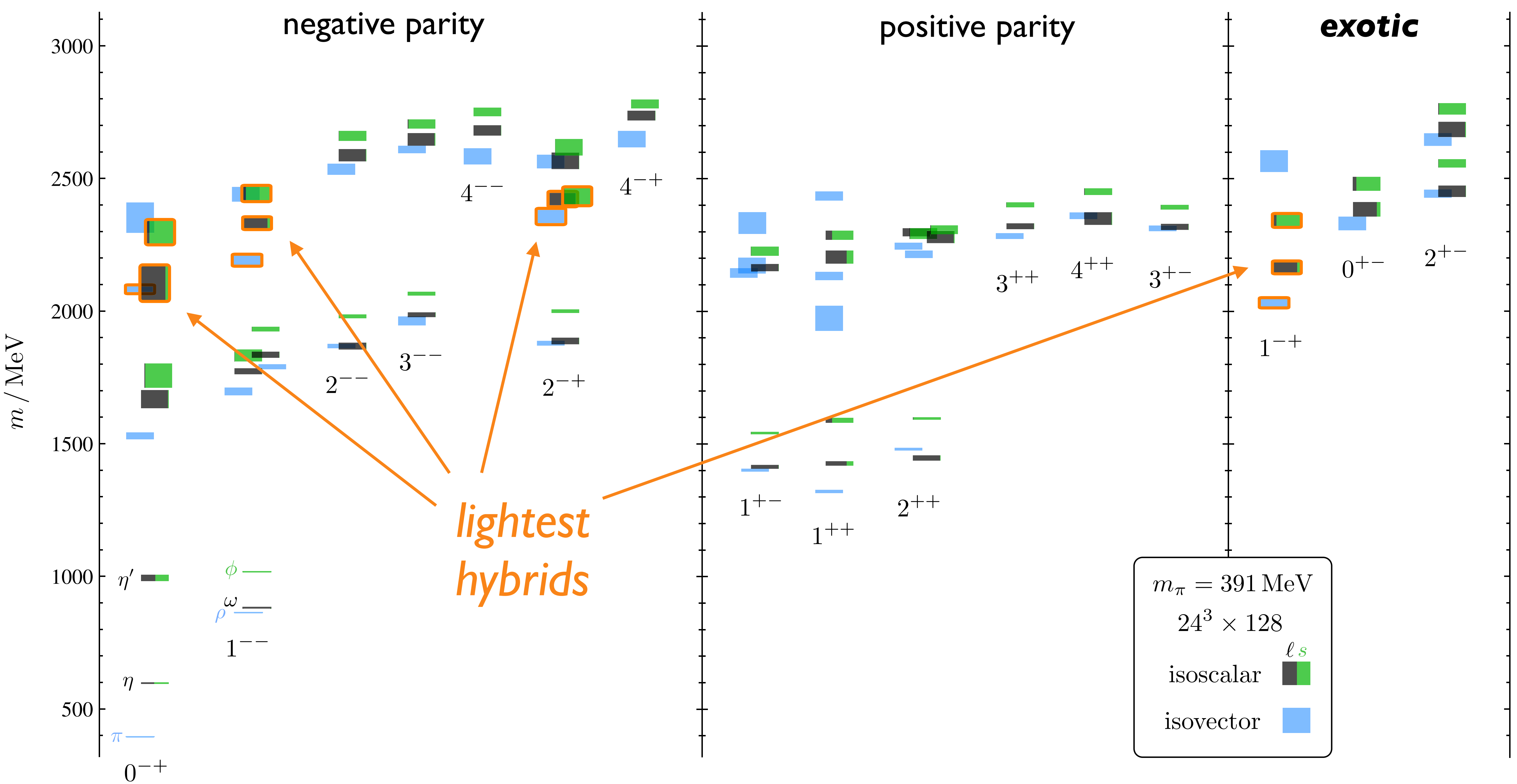
1^- nonet
neutrals: C=-

motivate the quark model ingredients from studying and classifying mesons!

fast forward 50 years...
and deploy some supercomputing to
use QCD to predict a meson spectrum

Meson Spectrum from Lattice QCD

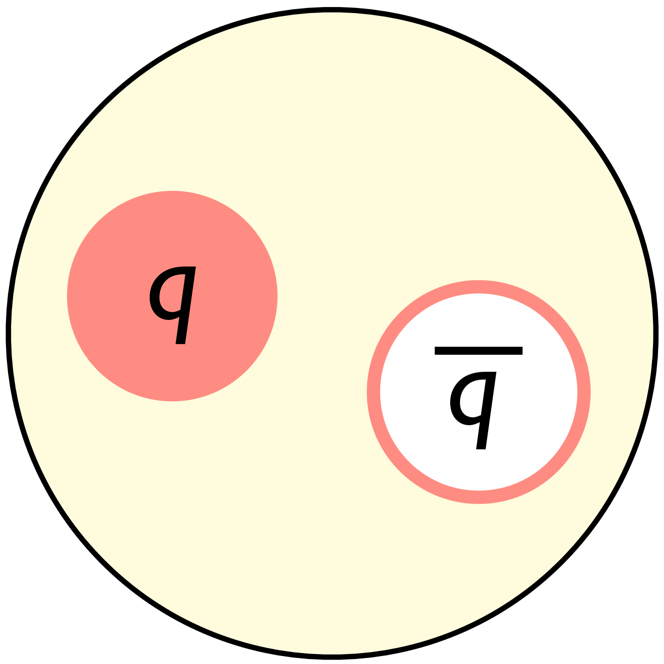
Dudek, Edwards, Guo, and Thomas, PRD 88, 094505 (2013)



All states have strangeness = 0

Hybrid Mesons

color singlet
quark anti-quark

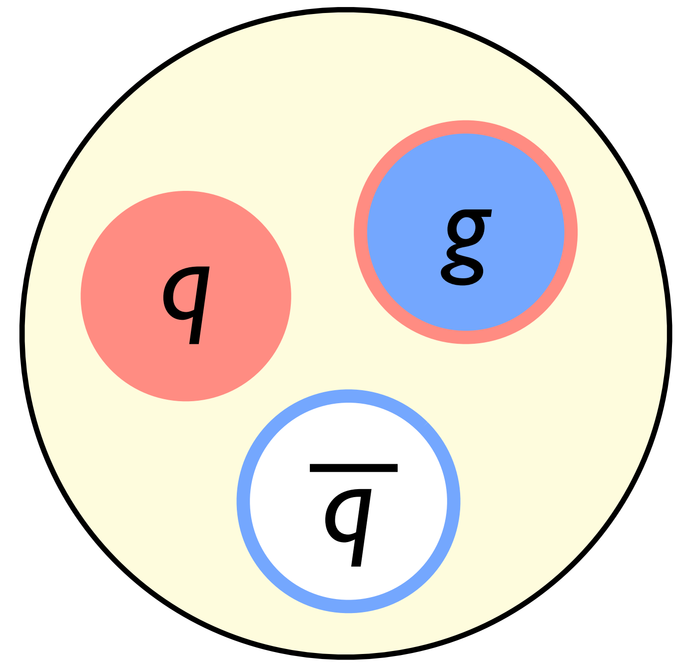


"gluonic contribution"

$(J^{PC})_g = 1^{+-}$

mass ≈ 1.0-1.5 GeV

color-octet
$q\bar{q}$ pair



$$\vec{J} = \vec{L} + \vec{S} \quad P = (-1)^{L+1} \quad C = (-1)^{L+S}$$

Allowed J^{PC}: $0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 2^{++}, \ldots$

Forbidden J^{PC}: $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \ldots$

Lightest Hybrids

	$S_{q\bar{q}} = 1$	$S_{q\bar{q}} = 0$
J^{PC}:	$0^{-+}, 1^{-+}, 2^{-+}$	1^{--}

"exotic hybrid"

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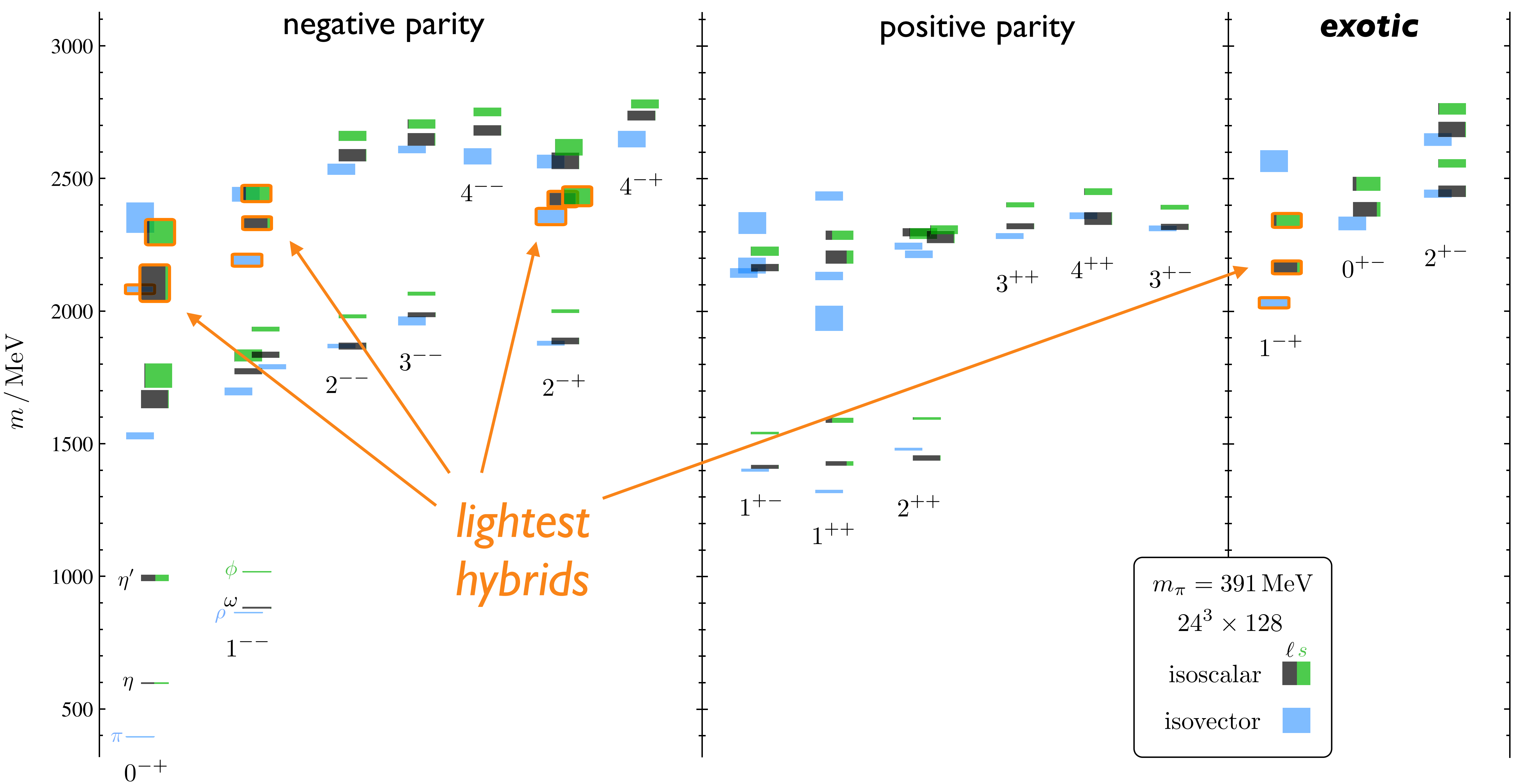
Recap

- Spectroscopy depends on identifying properties that are useful for sorting and classifying hadrons
  - Enables identification of states that don't fit a pattern
  - New patterns suggest new degrees of freedom
- QCD predicts new states that should not fit the standard patterns predicted by the quark-antiquark model of mesons
  - How do we produce them?
  - How do we detect them?
    - How do we measure the properties of mesons that we want to use to sort the spectrum?

Some Experimental Preliminaries

Meson Spectrum from Lattice QCD

Dudek, Edwards, Guo, and Thomas, PRD 88, 094505 (2013)



All states have strangeness = 0

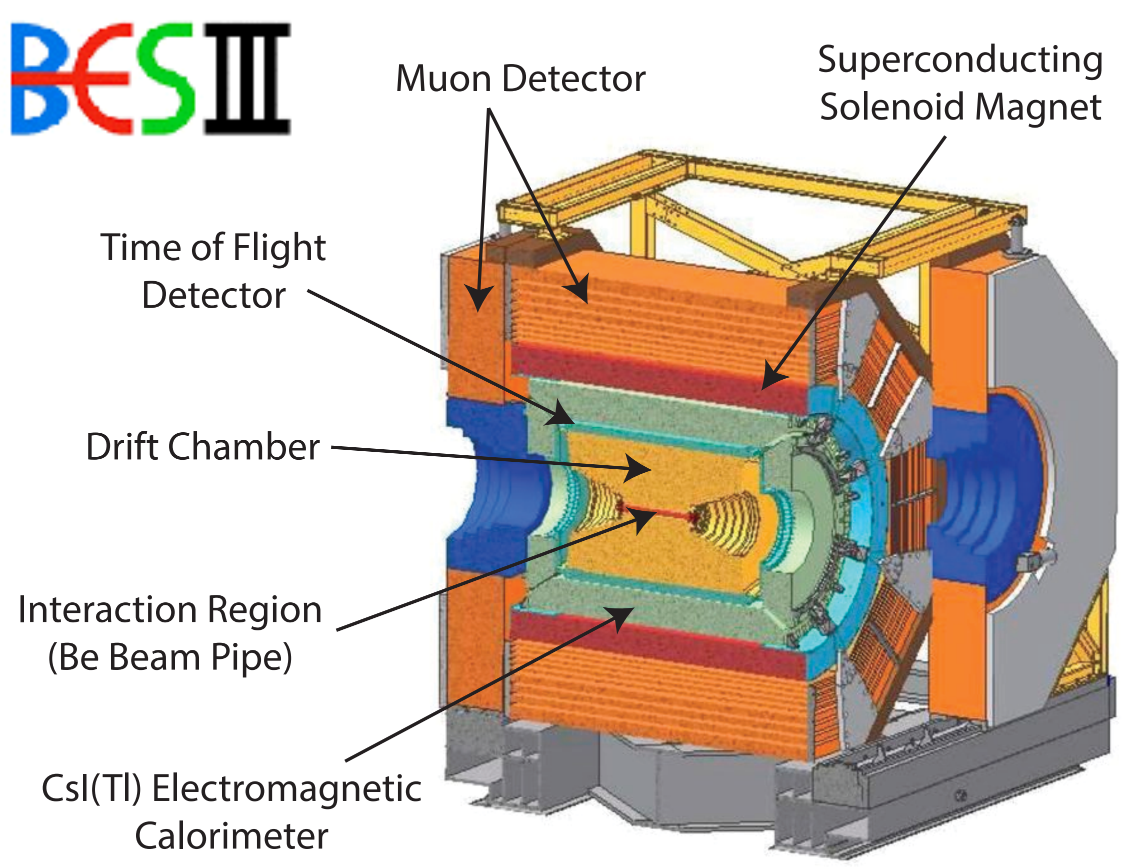
Decays and Conservation Laws

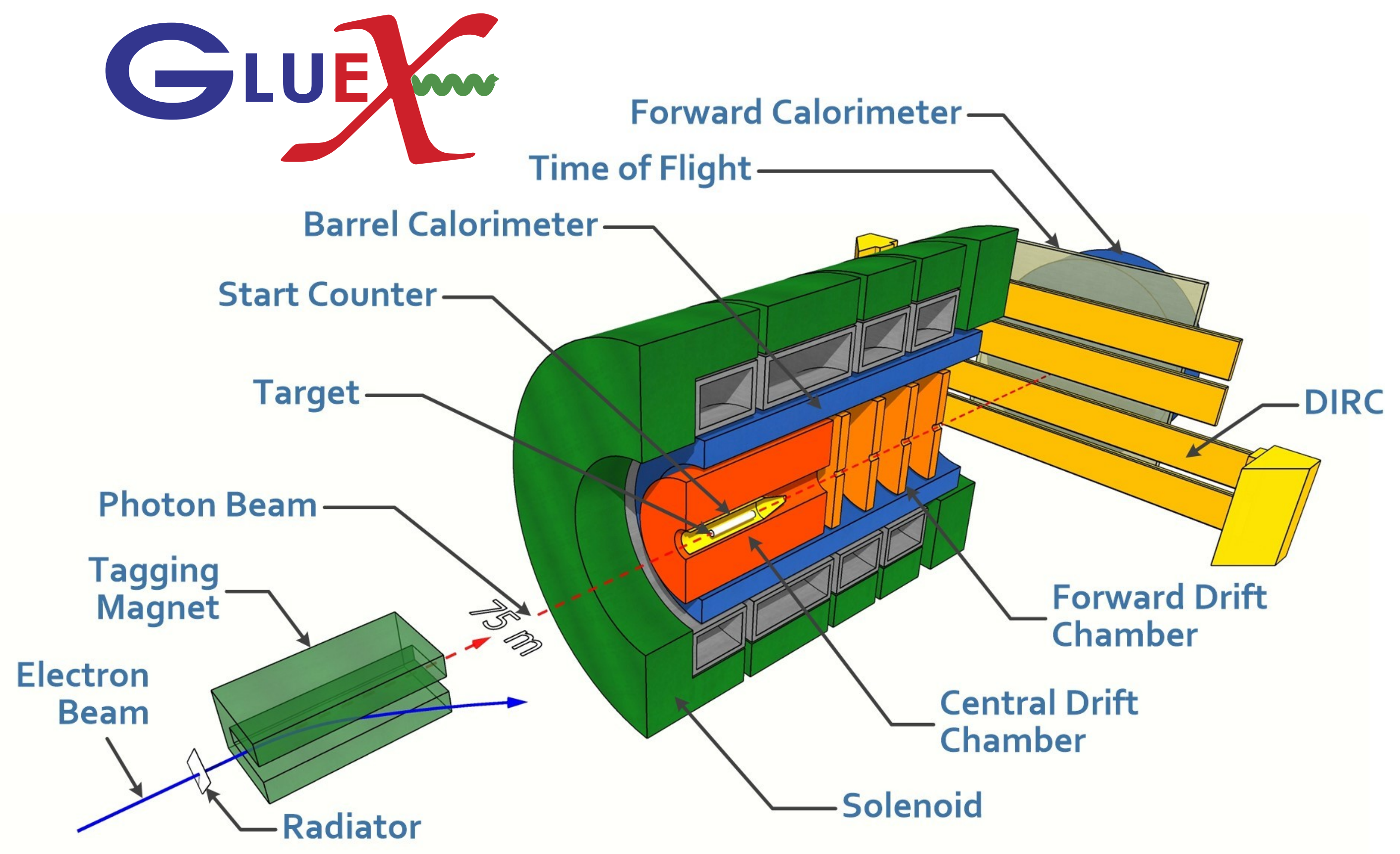
- Conservation laws that apply to all decays
  - angular momentum
  - four-momentum
  - charge
- Symmetries/conservations laws of strong interactions
  - C
  - P
  - isospin (mostly)
  - quark flavor: strangeness or charmness
- Measuring these properties for decay products directly informs us of the properties of the parent particle

Production and Detection

<u>Colliding Beam</u>
e^+e^-
proton-proton
proton-antiproton

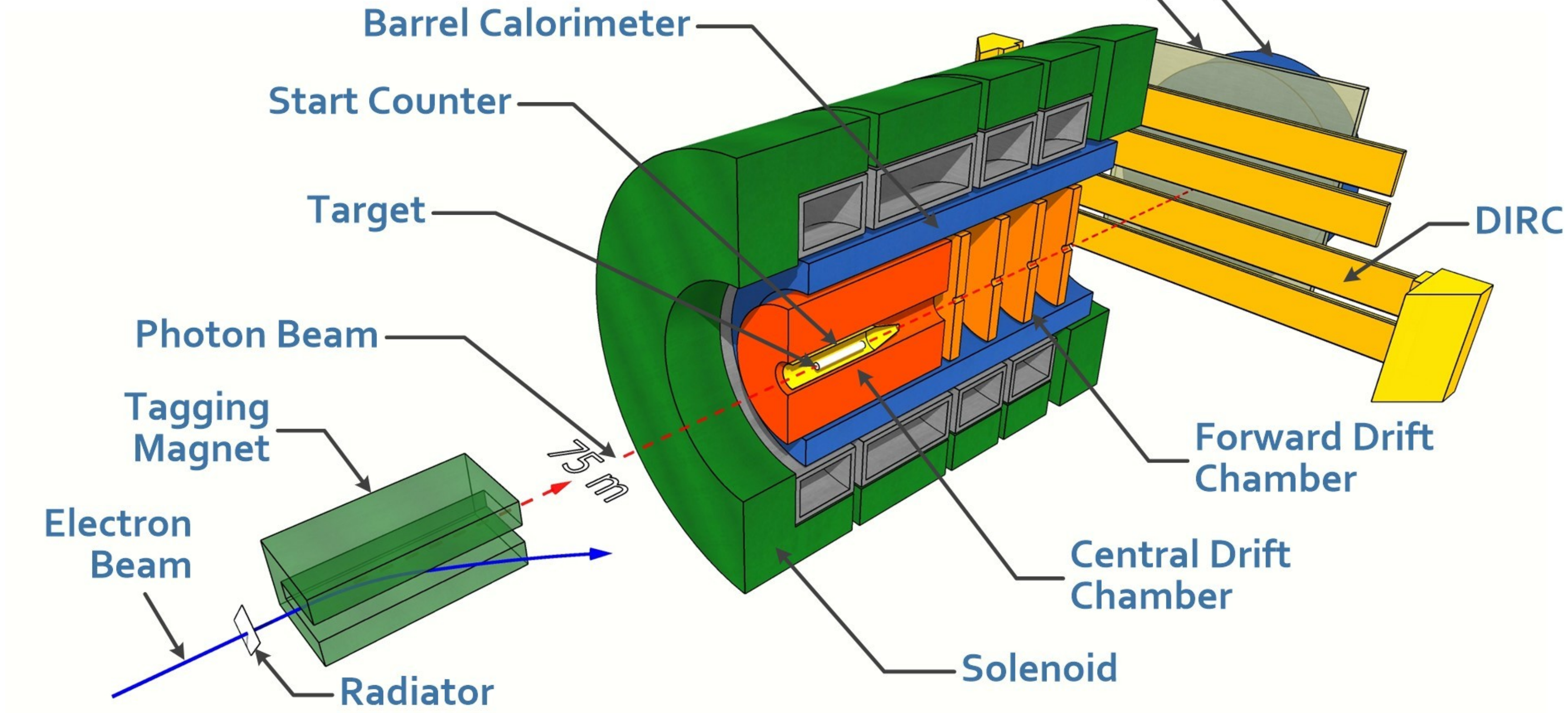
<u>Fixed Target</u>
electromagnetic or
hadron beams





Detection: Observables

- Long lived particles
  - charged: p, π, K
  - neutral: n, γ, K_L
- Types of detectors:
  - tracking: measure momentum
  - calorimetry: measure energy
  - particle ID: measure velocity
- Assemble pieces to get four-momentum



for a few GlueX γp collision events, see:
https://hpg.sitehost.iu.edu/hdvis/event.html

Histograms: Invariant Mass

$$\gamma p \to X$$

reconstruct all particles

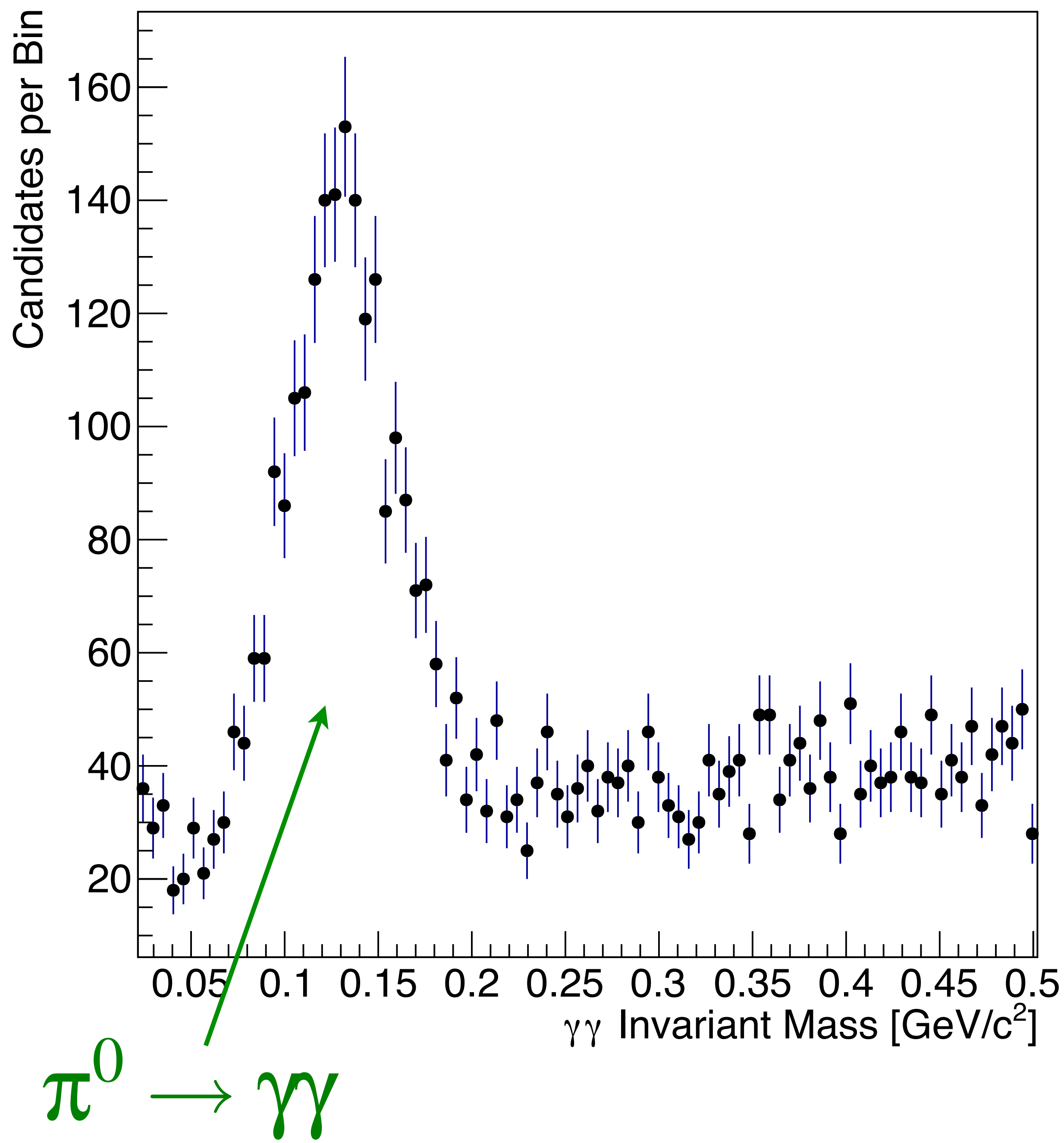
consider all combinations
of two photons

Homework -- show:

$$M^2_{\gamma\gamma} = |p_{\gamma,1} + p_{\gamma,2}|^2$$

$$M^2_{\gamma\gamma} = 2E_1E_2(1 - \cos\theta)$$

θ : angle between photon momenta



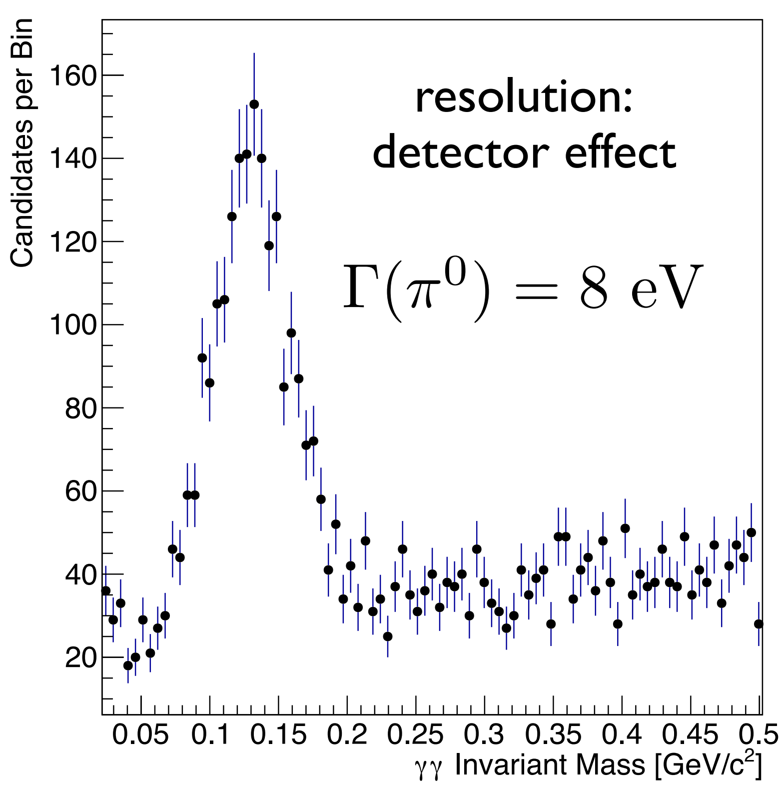
Branching Fractions and Widths

- Experimentally accessible:

$$B_i = \frac{\Gamma_i}{\Gamma_{\text{tot}}}$$

- Theoretically interesting:

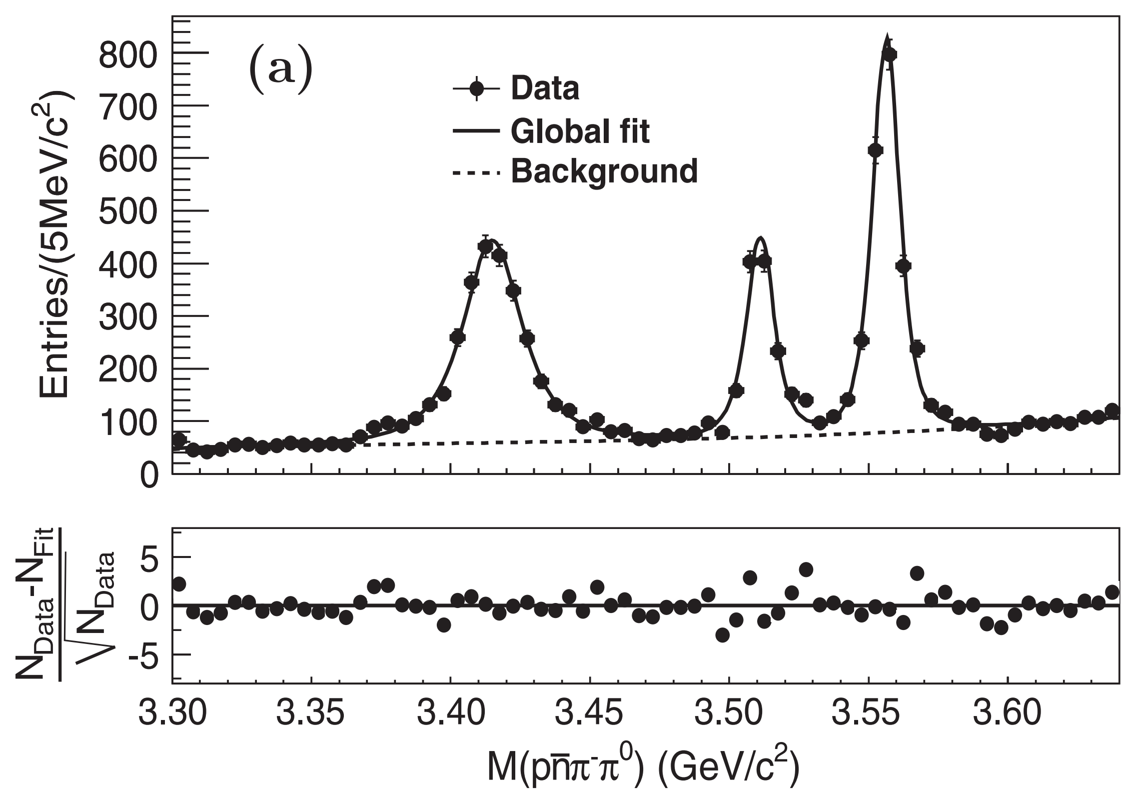
$$\Gamma_i \propto |\mathcal{M}_i|^2 \times (\text{phase space})$$



resolution:
detector effect

$$\Gamma(\pi^0) = 8 \text{ eV}$$

physics induced:
χ_{c0} is more likely to decay



PRD 86, 052011

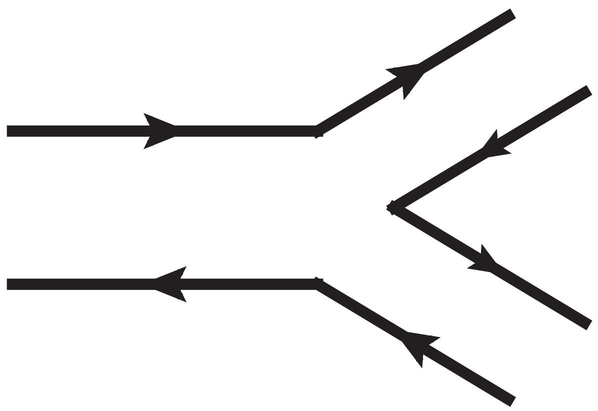
$$\Gamma(\chi_{c0}) = 10 \text{ MeV}$$

$$\Gamma(\chi_{c1}) = 0.8 \text{ MeV}$$

$$\Gamma(\chi_{c2}) = 1.9 \text{ MeV}$$

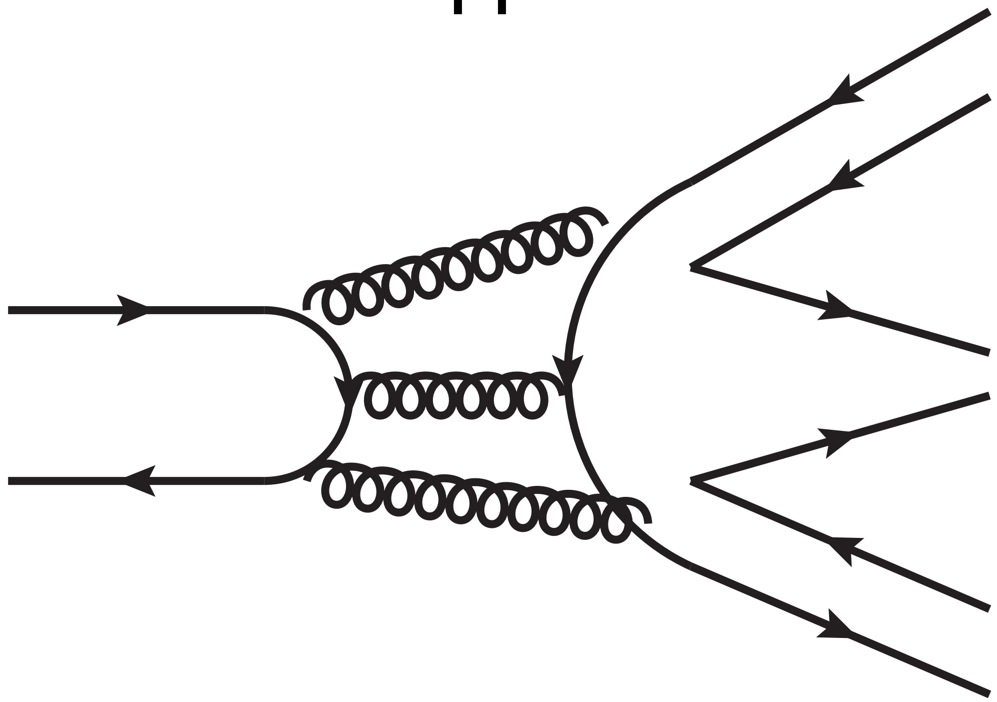
Decays: The OZI Rule
(S. Okubo, G. Zweig, and J. Iizuka)

OZI Favored



$$B(\phi \to KK) = 0.83$$

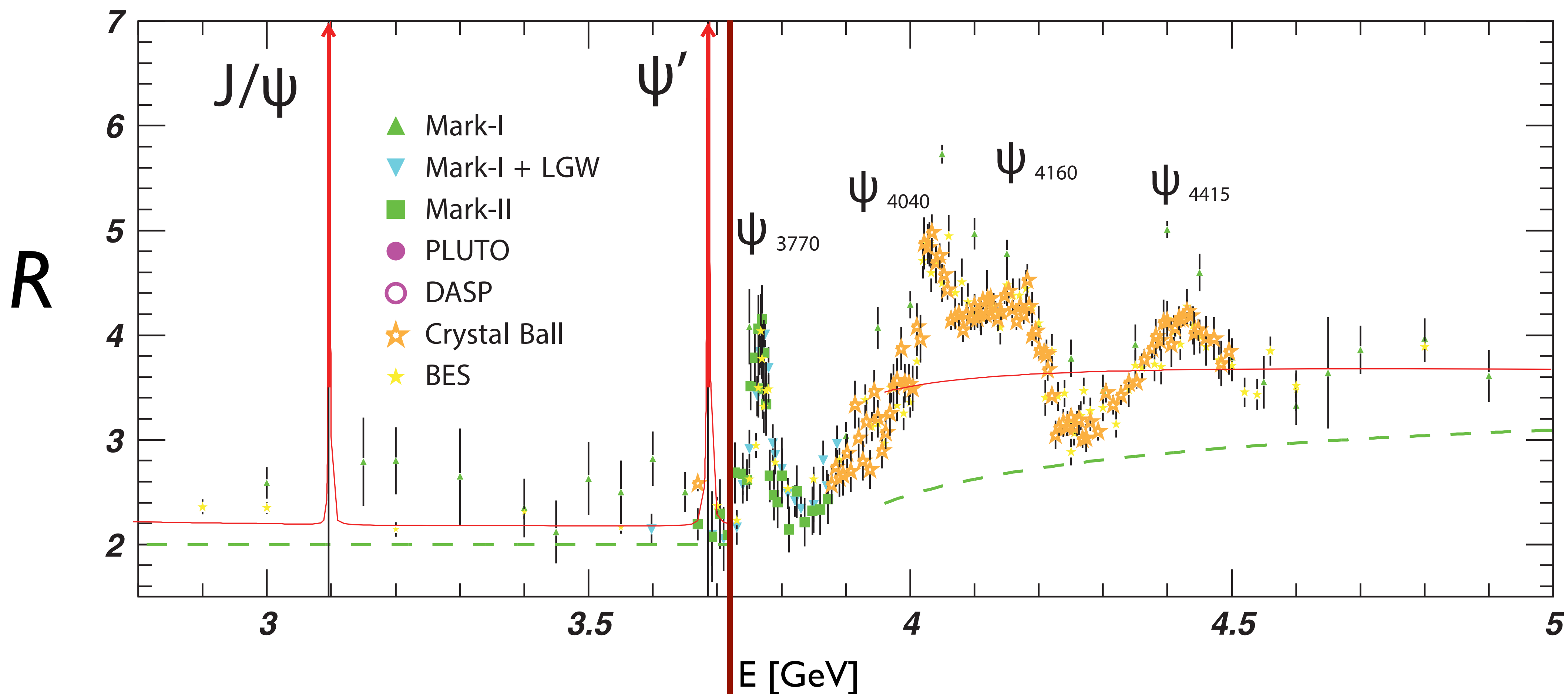
OZI Suppressed

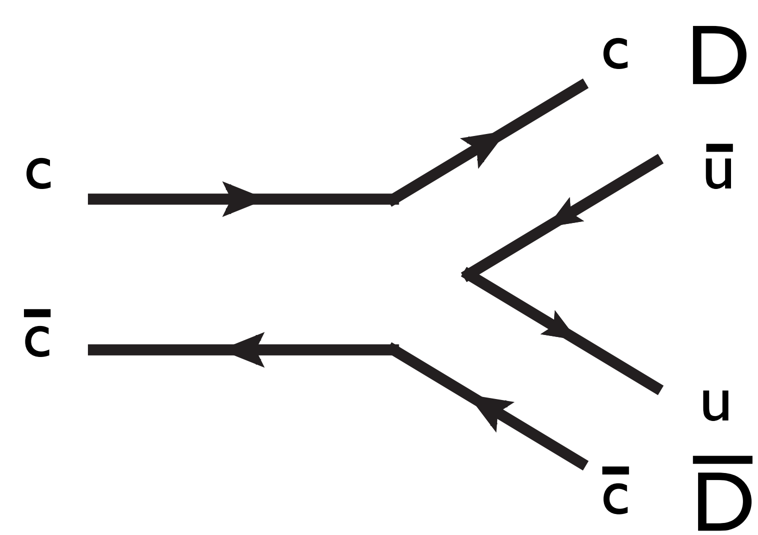


$$B(\phi \to \pi^+\pi^-\pi^0) = 0.15$$

Helps one infer "hidden" quark flavor of mesons.

OZI Rule and Widths





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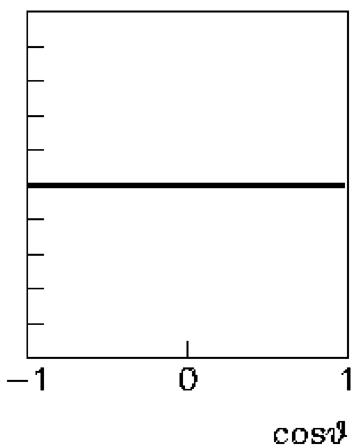
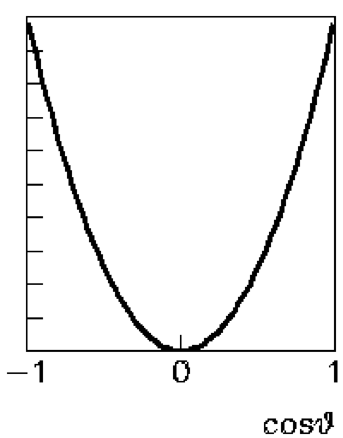
An Example: Measuring Spin

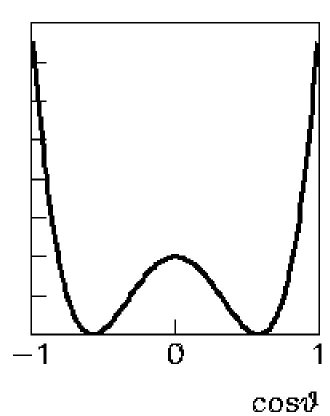
$\left|Y_0^0\right|^2$



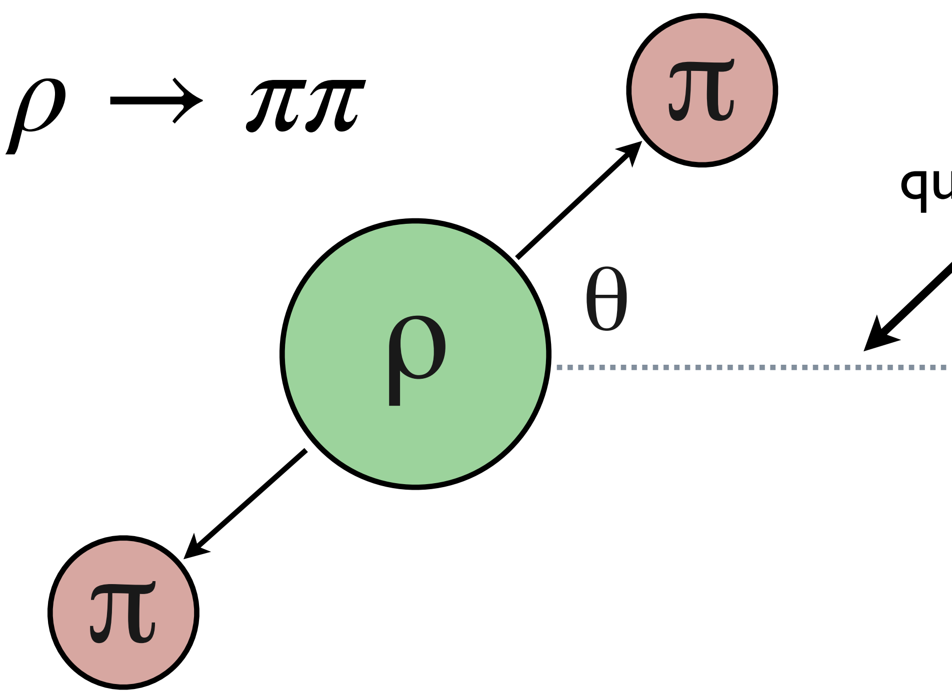
$\left|Y_1^0\right|^2$



$\left|Y_2^0\right|^2$

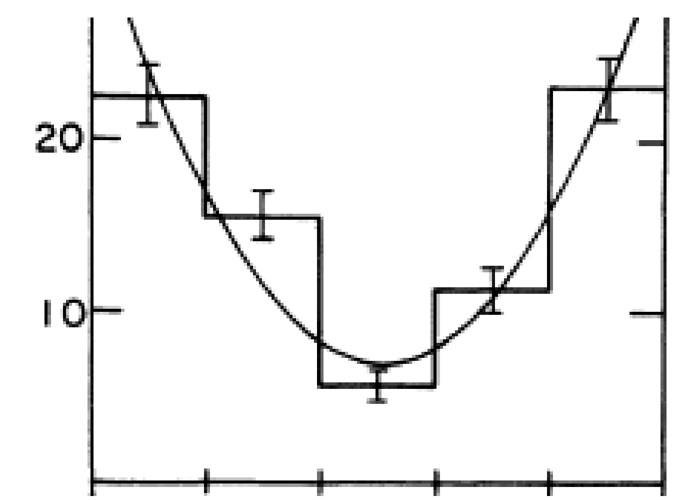


$\rho \rightarrow \pi\pi$



Pions are spinless so spin of ρ is carried in the orbital angular momentum of the two pions.

From data conclude J = 1



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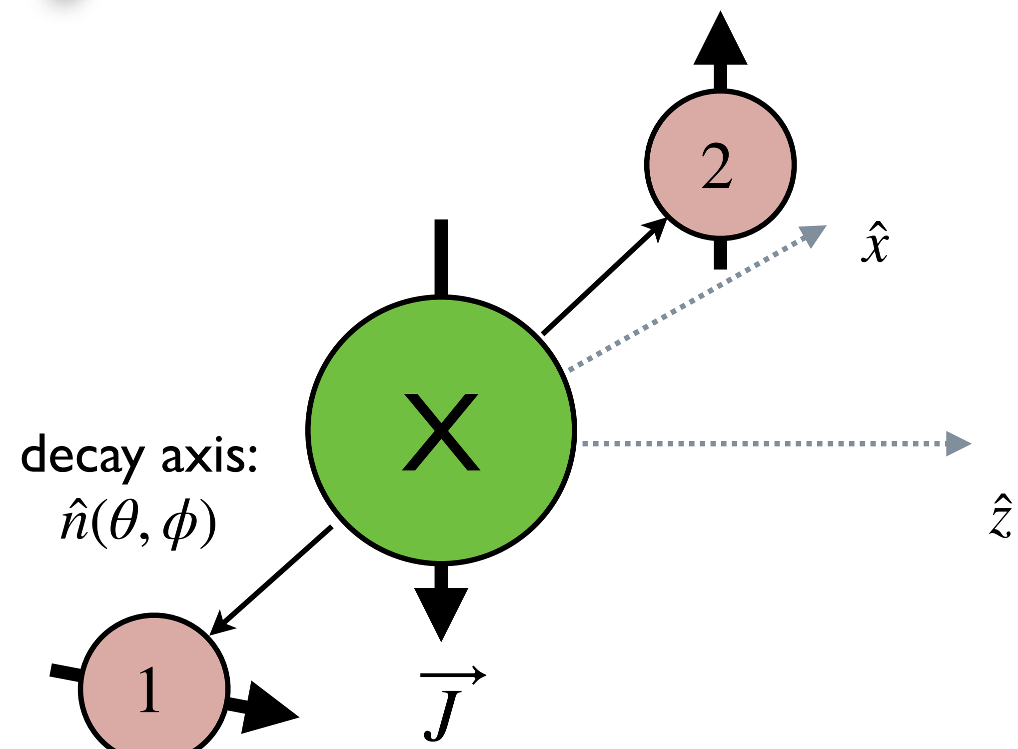
DIFFERENTIAL π-π CROSS SECTIONS: EVIDENCE FOR THE SPIN OF THE ρ MESON*

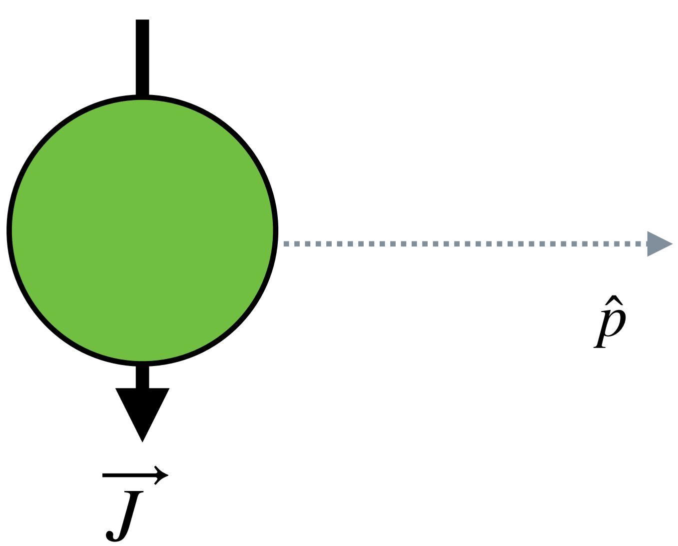
D. Duane Carmony† and Remy T. Van de Walle‡
Lawrence Radiation Laboratory, University of California, Berkeley, California

(Received November 6, 1961; revised manuscript received December 27, 1961)

Helicity

- Helicity operator is the spin projection along the momentum of the particle
  - $h = \overrightarrow{J} \cdot \hat{p}$
  - eigenvalues: $\lambda = -S, \ldots, S$
  - invariant under rotation
- Helicities of stable final state particles are not measured and must be summed over when computing decay distributions





$X \rightarrow 1 + 2$

initial state X: $|Jm\rangle$

two-particle final state: $|\theta, \phi, \lambda_1, \lambda_2\rangle$

decay amplitude $\propto D^{J*}_{M,\lambda}(\phi, \theta, -\phi) A_{\lambda_1 \lambda_2}$

where: $\lambda = \lambda_1 - \lambda_2$

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NNPSS
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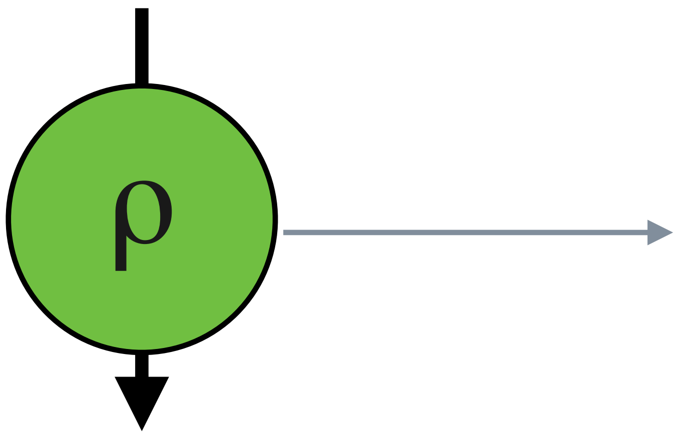
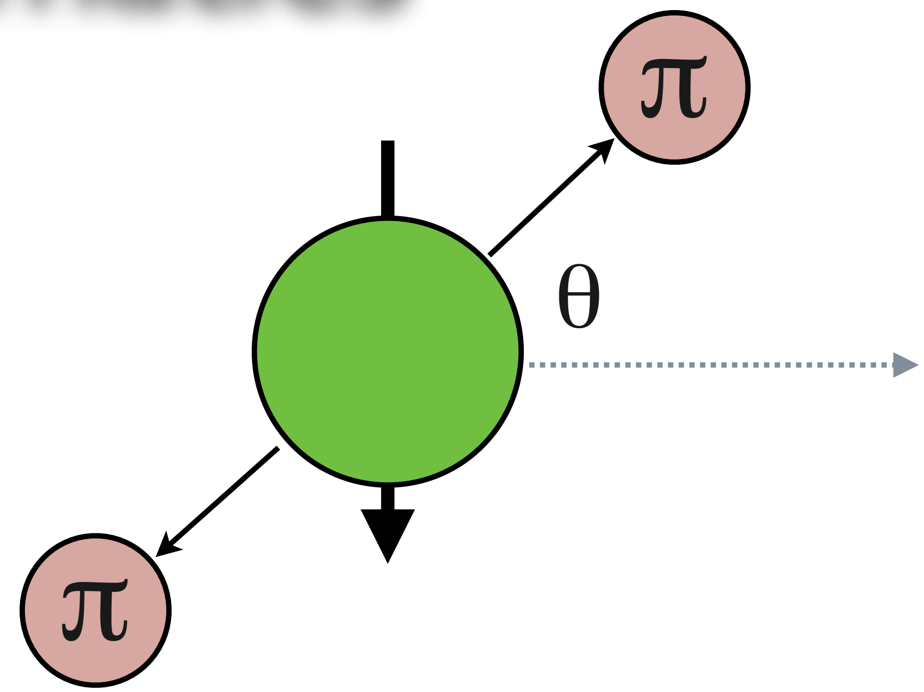
Decay Kinematics





initial ρ configuration
in the helicity frame

$$J = 1,\ m = 0$$

spinless pions
each have helicity zero
so the helicity sum is zero
and in this frame: $m' = 0$

Rotation between frames given by

$$D^J_{m',m}(\alpha, \beta, \gamma) = e^{-im'\alpha} d^J_{m',m}(\beta) e^{-im\gamma}$$

$$d^J_{m',m} = (-1)^{m-m'} d^J_{m,m'} = d^J_{-m,-m'}$$

For the sketch above: $\alpha = \gamma = 0$ and $\beta = \theta$

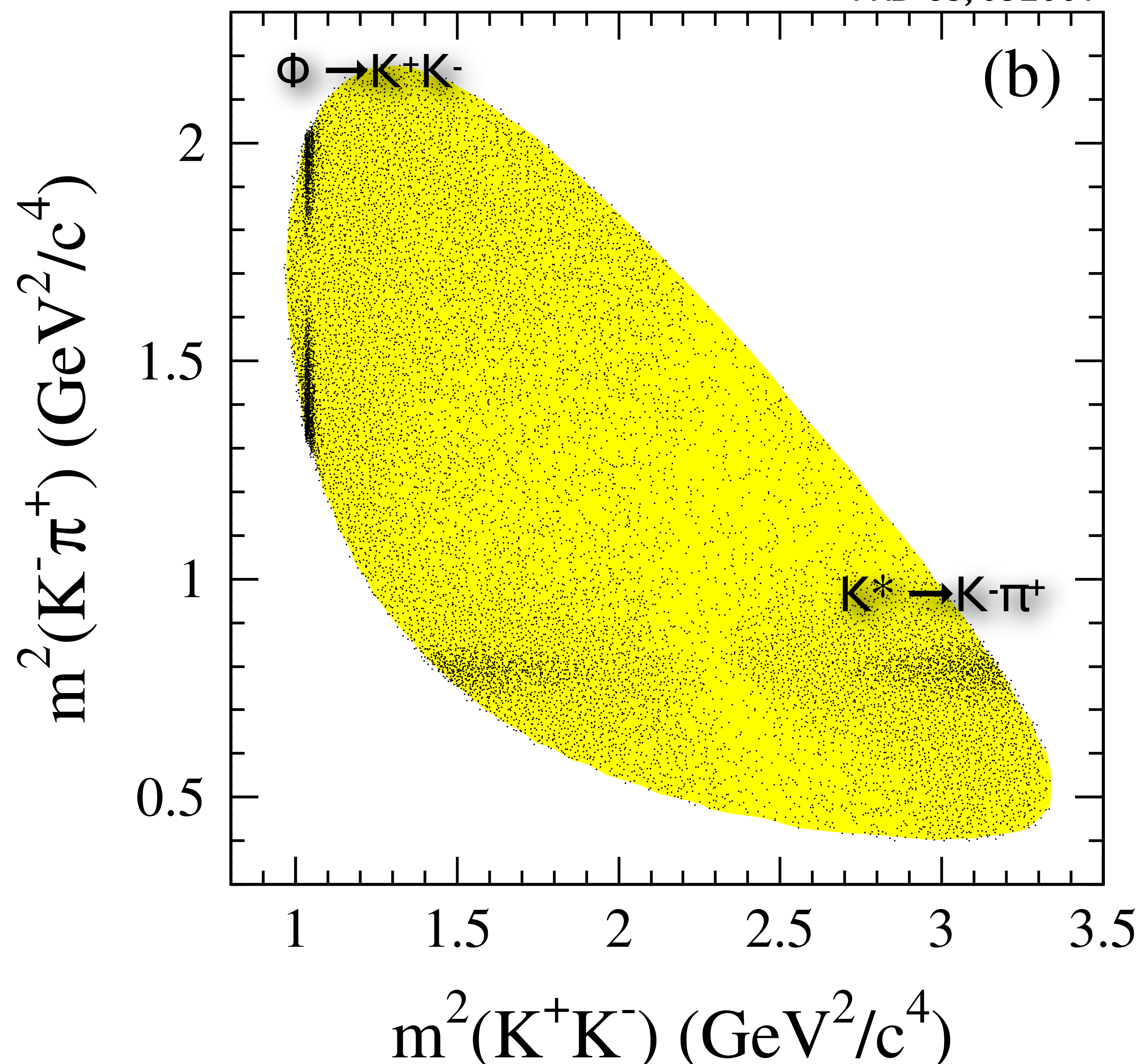
$$\boxed{d^1_{0,0}(\theta) = \cos\theta} \qquad d^1_{1,0}(\theta) = \frac{-\sin\theta}{\sqrt{2}}$$

Dalitz Plots

- spinless particle → 3 spinless particles: X → 1 2 3
- $M_X^2 = M_{12}^2 + M_{23}^2 + M_{13}^2$
  - for any X, dynamics is a function of two variables: M_{12} and M_{23}
- All information about decay can be learned by studying a Dalitz plot of M_{12}^2 vs. M_{23}^2
  - phase space is uniform on this plot

$D_s^+ \rightarrow K^+K^-\pi^+$

PRD 83, 052001



Dalitz Plots

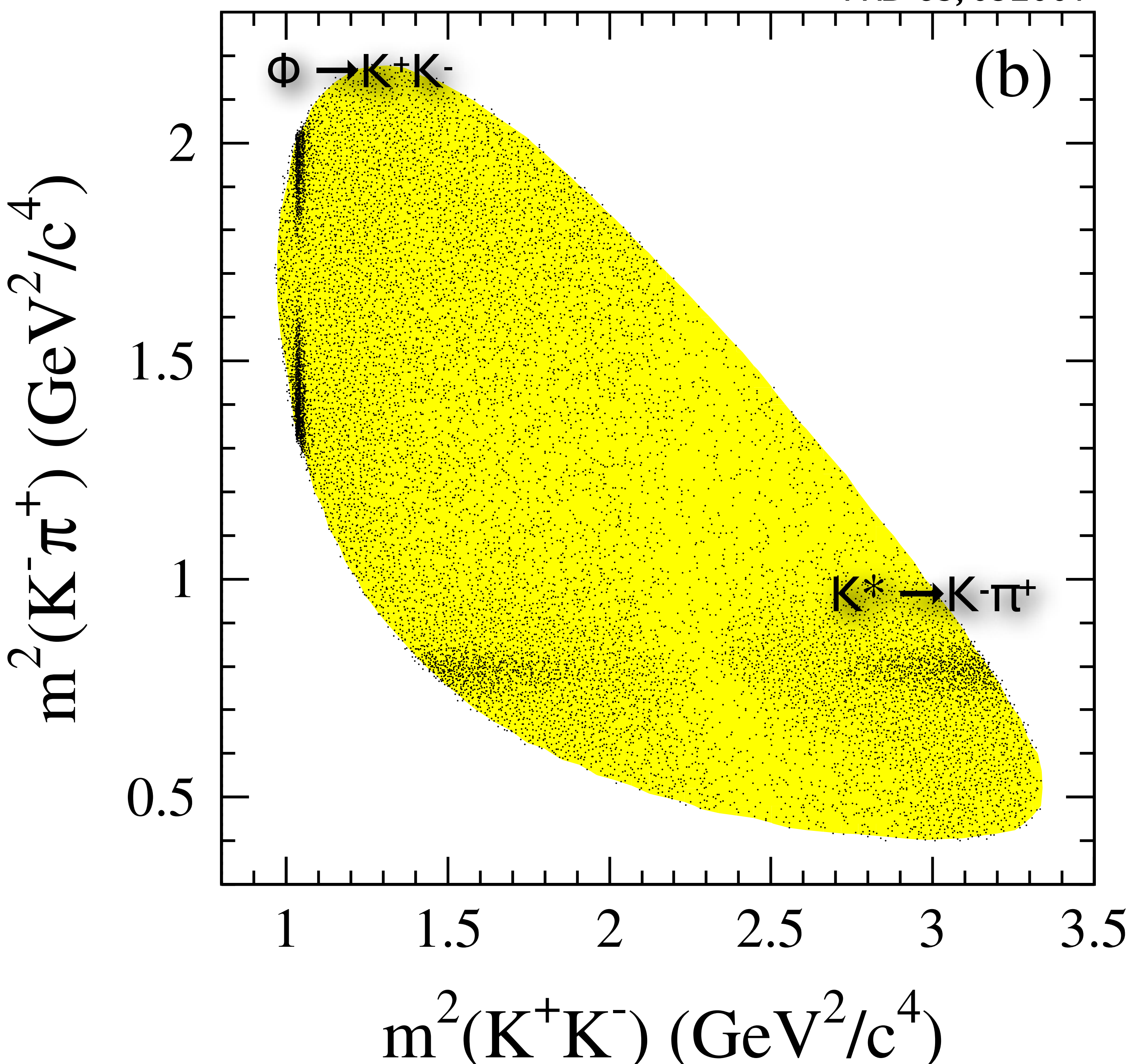
Homework: the decay of a spinless particle to 3 spinless particles can be described by 3 x 4-vectors = 12 numbers.

Use symmetry arguments and conservation laws to show that 10 of the 12 unknowns can be eliminated leaving only two remaining variables to describe the physics of the decay.

Any two variables will work, the Dalitz plot is a common choice.

$D_s^+ \to K^+K^-\pi^+$

PRD 83, 052001



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Experimental Strategy

- Search for new particles
  - bumps in invariant mass spectra
  - unique decay patterns (angular distributions) in phase space
- Use experiment to determine properties of mesons:
  - mass and width
  - decay modes
  - quantum numbers: J^{PC}
- Patterns of mesons then test predictions of the hadron spectrum from models or direct calculations of QCD

Summary

- QCD exhibits interesting behavior at the low-energy/long-distance limit: strong gluon-gluon interactions and confinement
- How do hadrons and their properties emerge from the underlying theory of QCD?
  - Are there particles like hybrid mesons or glueballs that would be unique manifestations of the gluonic degrees of freedom in QCD?
- Spectroscopy provides insight into the fundamental structure of hadrons, and can do so over both the long and short distance regimes of QCD.
- A wide variety of experiments can be used to produce and study hadrons: properties relevant for spectroscopy such as mass, width and J^{PC} can be inferred from experimental data