Fundamental Symmetries

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Outline :

1. Introduction and Motivation

2. The Standard Model

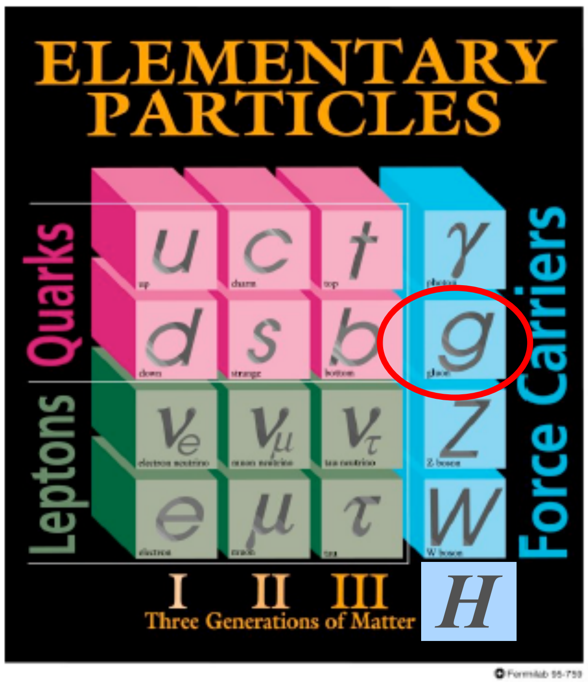
3. Selected examples
   1. $\eta \rightarrow 3\pi$ and light quark mass ratio
   2. Anomalous magnetic moment of the muon
   3. Axial form factor of the nucleon and neutrino physics

4. Conclusion and outlook

2.4 Strong Interactions

Introduction

- In particle physics a simpler table made of leptons and quarks: the degrees of freedom



Charge: 0, -1 | +2/3, -1/3

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

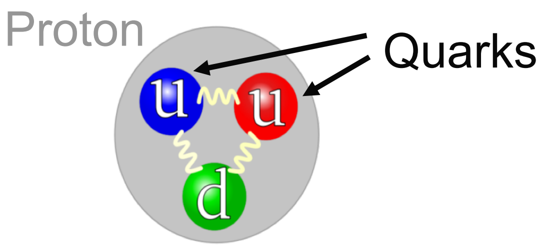
$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

Int.: w, w,e | w,e,s

- 3 forces: electromagnetic, weak and strong forces

Strong interaction

- Problem: quarks and gluons are bound inside hadrons



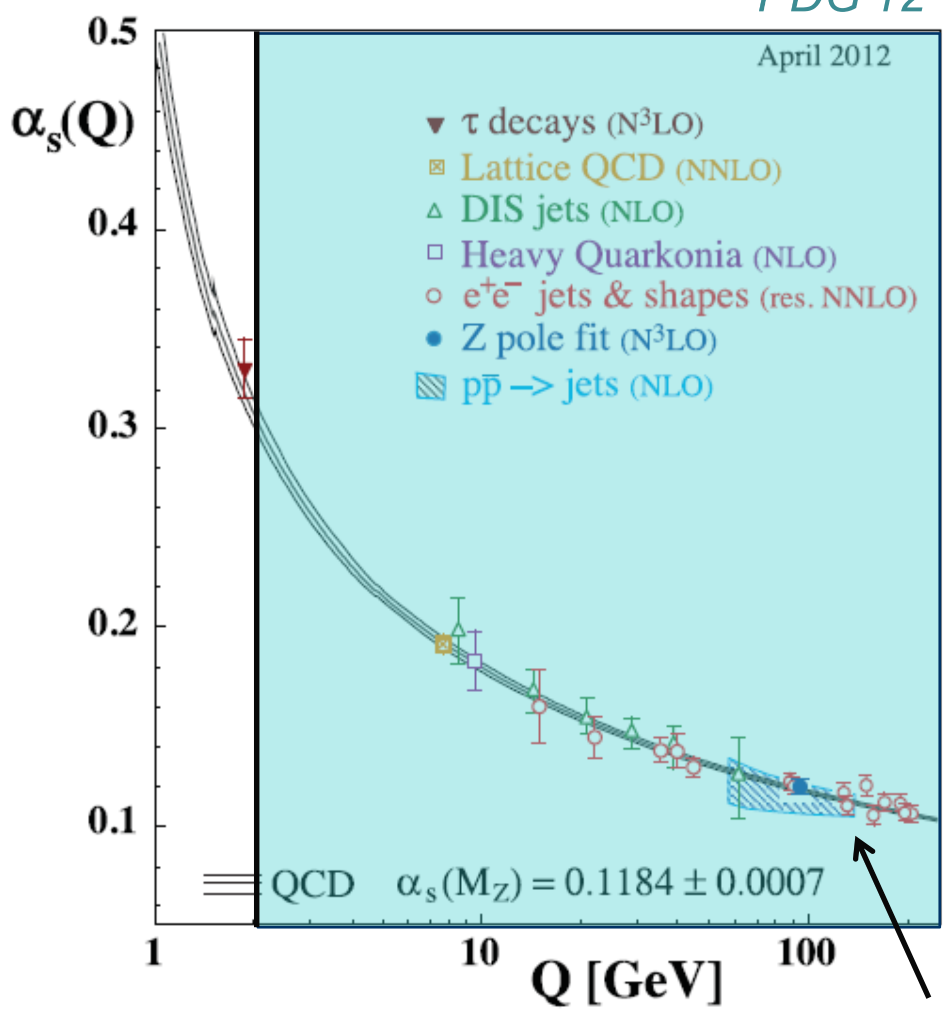
- High energies, short distance:
  α_S small ⇨ Asymptotic freedom

  Perturbative QCD
  Theory "easy" to solve

Order-by-order *expansion* in $\frac{\alpha_s(\mu)}{\pi}$

$$\sigma = \sigma_0 + \underbrace{\frac{\alpha_s}{\pi}\sigma_1}_{\text{small}} + \underbrace{\left(\frac{\alpha_s}{\pi}\right)^2 \sigma_2}_{\text{smaller}} + \left(\frac{\alpha_s}{\pi}\right)^3 \sigma_3 + \dots$$

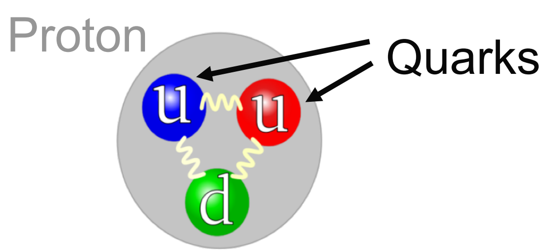
PDG'12



Asymptotic freedom

Strong interaction

- Looking for new physics in hadronic processes ⇨ not direct access to quarks due to confinement



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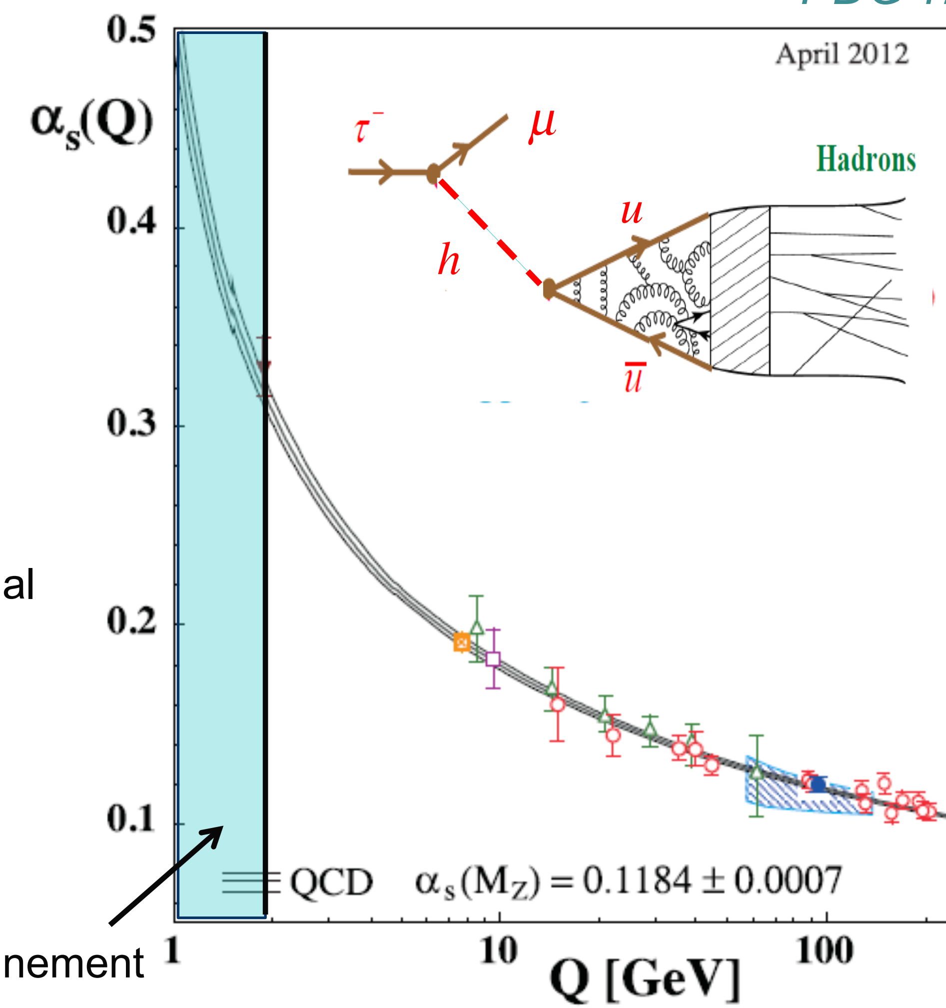

Confinement

➢ Low energy (Q <~1 GeV), long distance: *α_S becomes large !*

⇨ *Non-perturbative QCD*

A perturbative expansion in the usual sense fails

⇨ Use of alternative approaches, expansions…

Strong interaction

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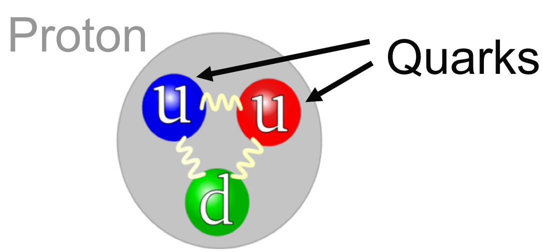

➢ Non-perturbative methods:
  - Numerical simulations on the lattice

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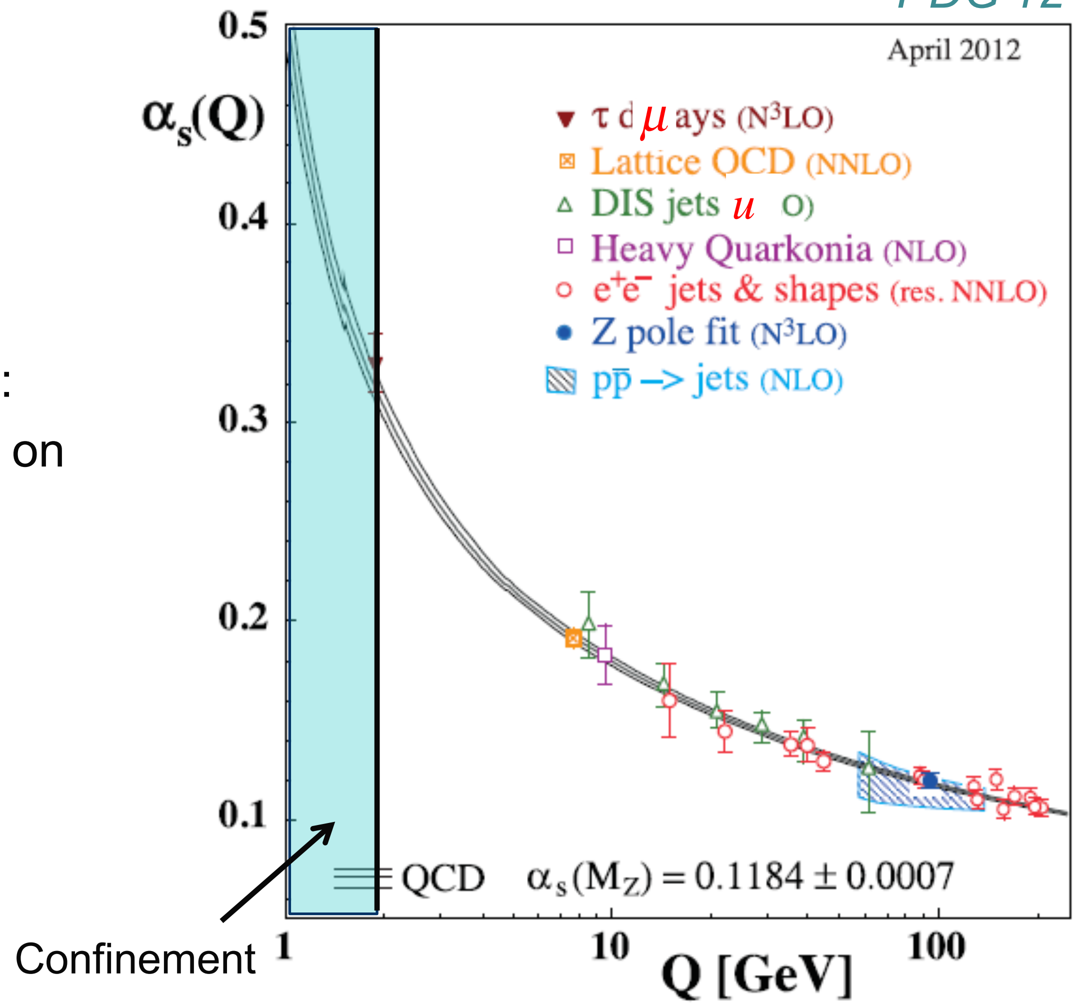
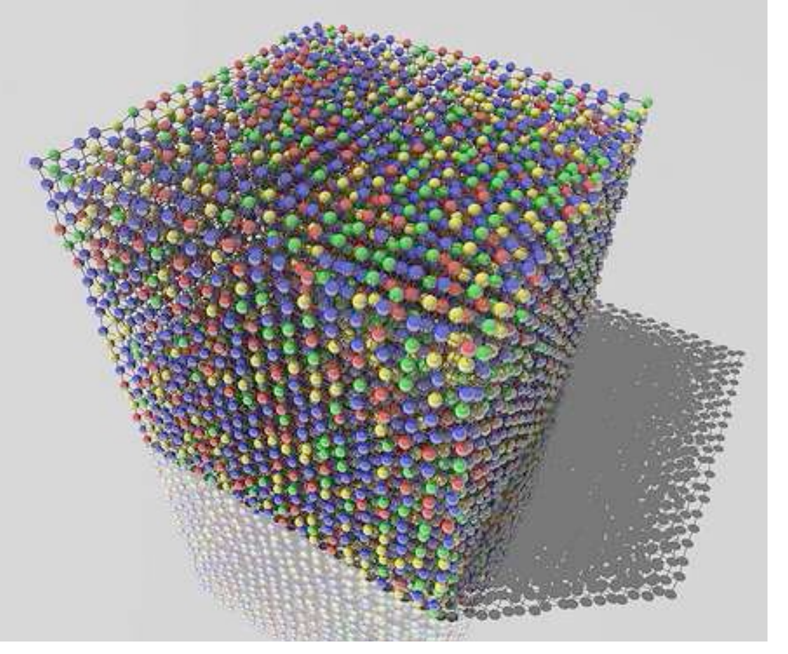


Lattice QCD

- Principle: Discretization of the space time and solve QCD on the lattice numerically
  - All quark and gluon fields of QCD on a 4D-lattice
  - Field configurations by Monte Carlo sampling



- Important subtleties due to the discretization, should come back to the continuum, formulation of the fermions on the lattice…

Strong interaction

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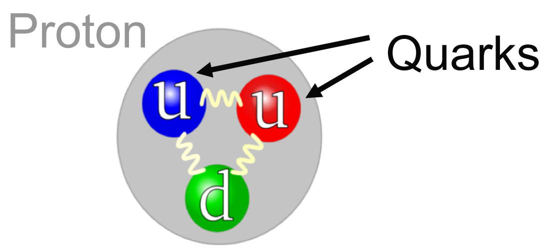
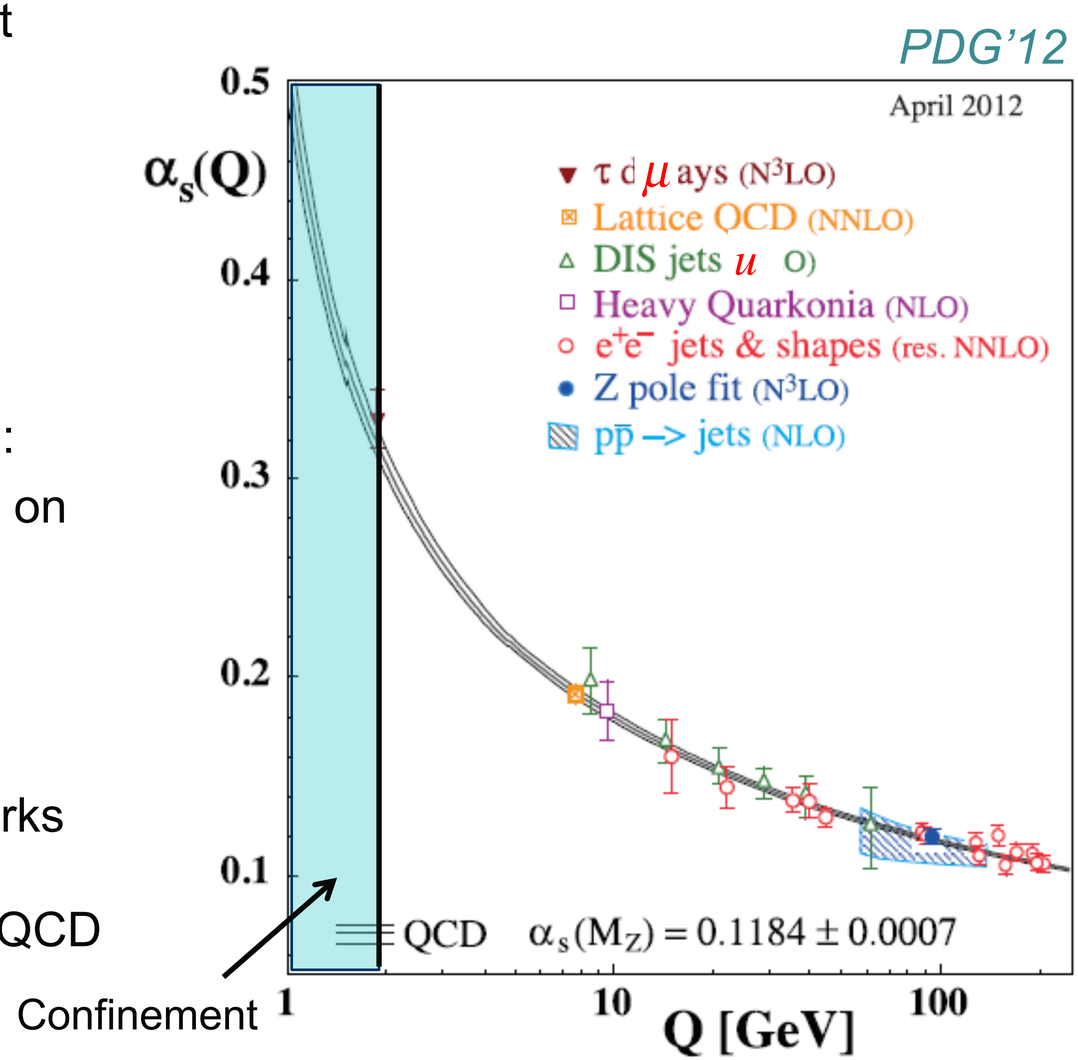


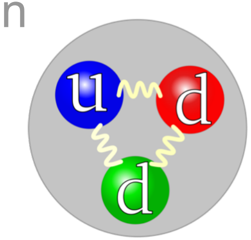


➢ Non-perturbative methods:
  - Numerical simulations on the lattice
  - Analytical methods:
    Effective field theory
    Ex: ChPT for light quarks
    Dispersion relations
    Synergies with lattice QCD

⇨ *Hadronic Physics*

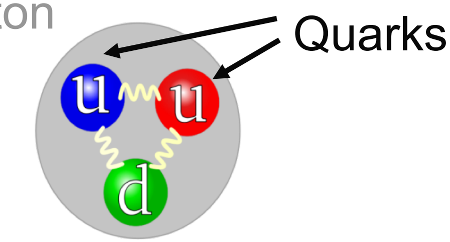
Quark masses

Neutron



vs.

Proton



$M_n = 939.57$ MeV

$M_p = 938.27$ MeV

- Strong force: If $m_u \sim m_d$: $M_n \sim M_p$ *isospin symmetry* *Heisenberg'60*

  Countless experiments have shown that strong force obeys isospin symmetry Results are the same if we interchange neutrons and protons (or up and down quarks)

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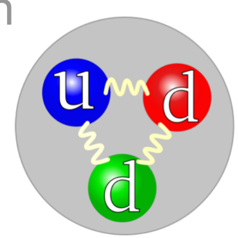

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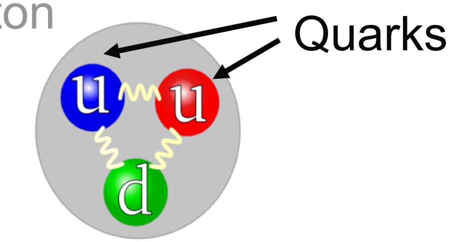

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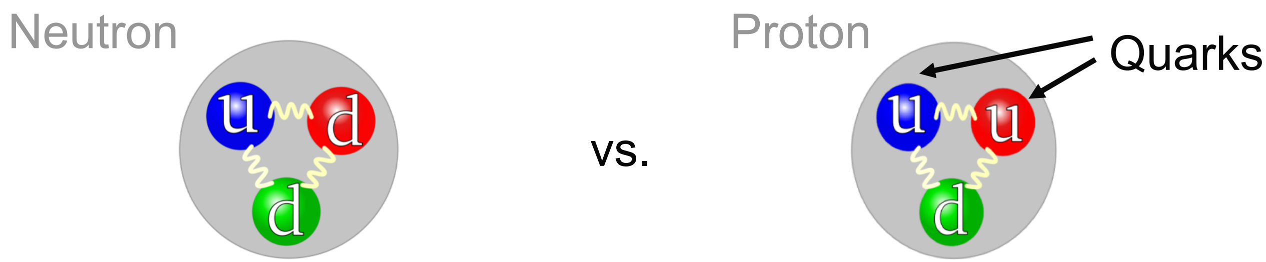

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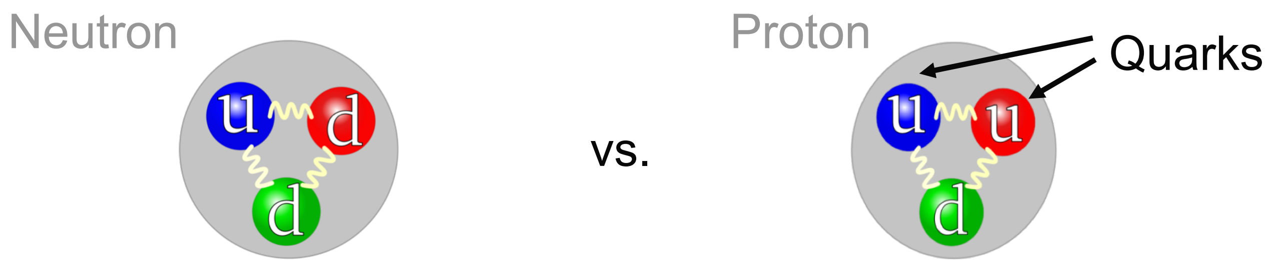
- Electromagnetic energy: one obvious difference between a neutron and a proton is their electric charges:

  $Q_P = 1$ and $Q_n = 0$ Since $E_e \propto \frac{Q^2}{R}$ ⇨ $M_p > M_n$?

Quark masses



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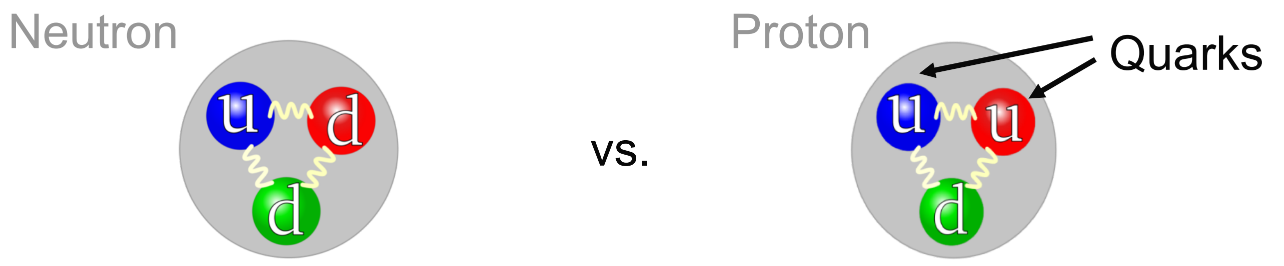
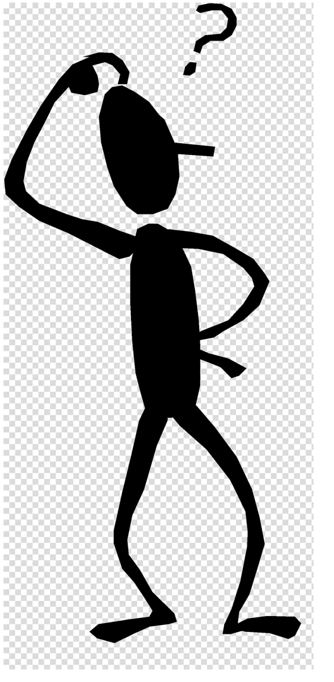
⇨ *Terrible consequences :* Proton would decay into neutrons and there will be no chemistry and we would not be there in this room!

Quark masses





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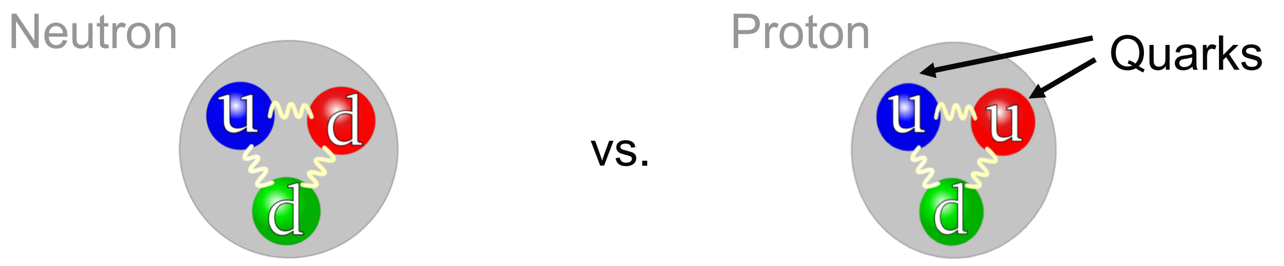

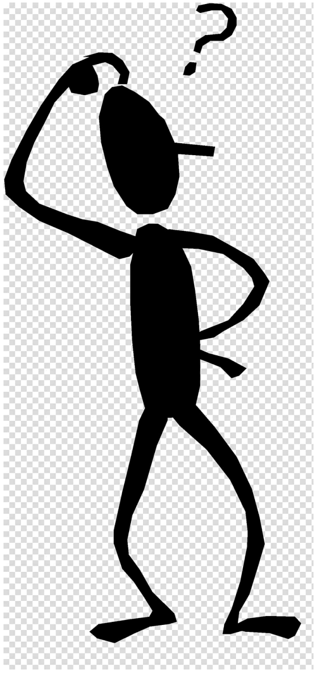

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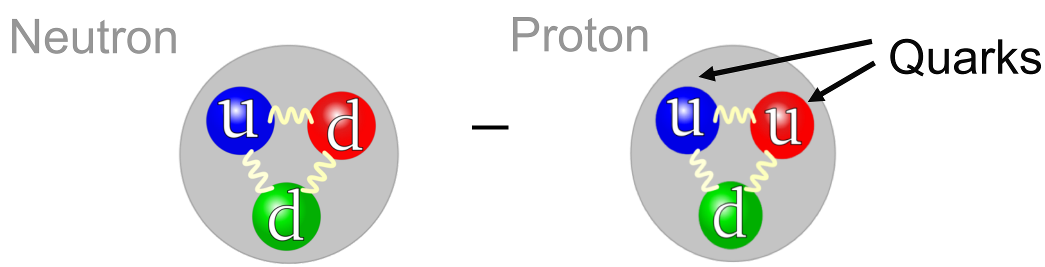
- Another small effect in addition to e.m. force:

  different fundamental quark masses $m_d \neq m_u$

  ⇨ Different coupling to Higgs field

Quark masses



QUARKS

The u-, d-, and s-quark masses are estimates of so-called "current-quark masses," in a mass-independent subtraction scheme such as $\overline{\text{MS}}$ at a scale $\mu \approx 2$ GeV. The c- and b-quark masses are the "running" masses in the $\overline{\text{MS}}$ scheme. For the b-quark we also quote the 1S mass. These can be different from the heavy quark masses obtained in potential models.

Particle Data Group'18

u $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

$m_u = 2.2^{+0.5}_{-0.4}$ MeV Charge $= \frac{2}{3}\ e$ $I_z = +\frac{1}{2}$

$m_u/m_d = 0.48^{+0.07}_{-0.08}$

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d $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

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$m_s/m_d = 17$–22

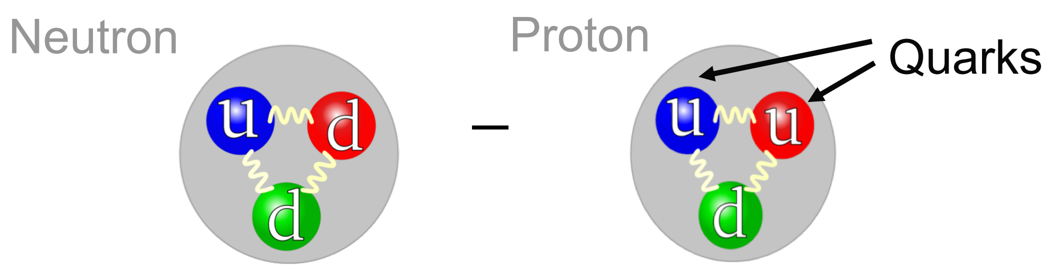
$\overline{m} = (m_u + m_d)/2 = 3.5^{+0.5}_{-0.2}$ MeV

$$m_d - m_u = 4.7 - 2.2 = 2.5 \text{ MeV}$$

Quark mass difference more important than e.m. effect

Neutrons can decay in protons!

Quark masses



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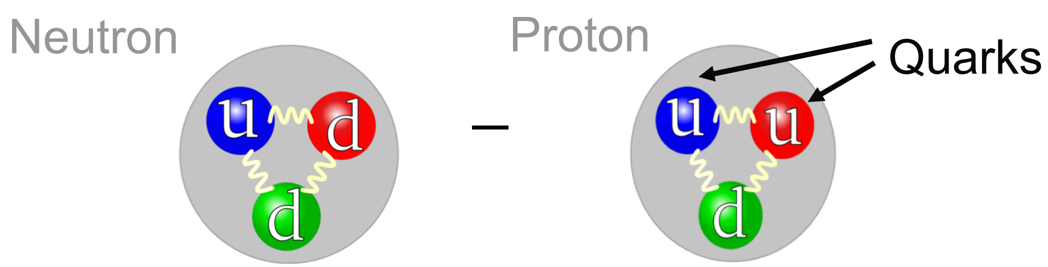
Neutron lifetime experiments

Quark masses



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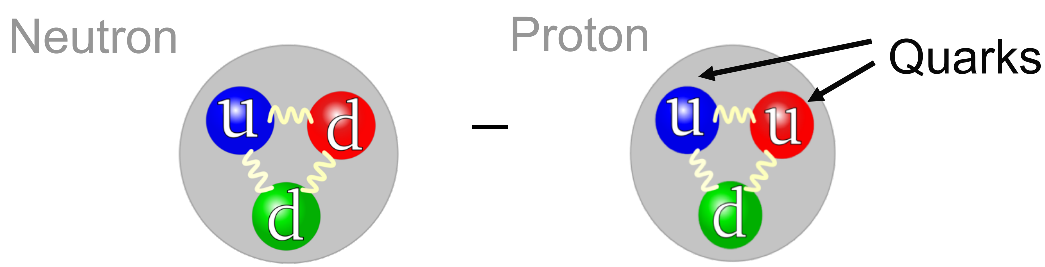
To determine these fundamental parameters need to know how to disentangle them from *QCD*

⇒ treat *strong interactions*

Quark masses



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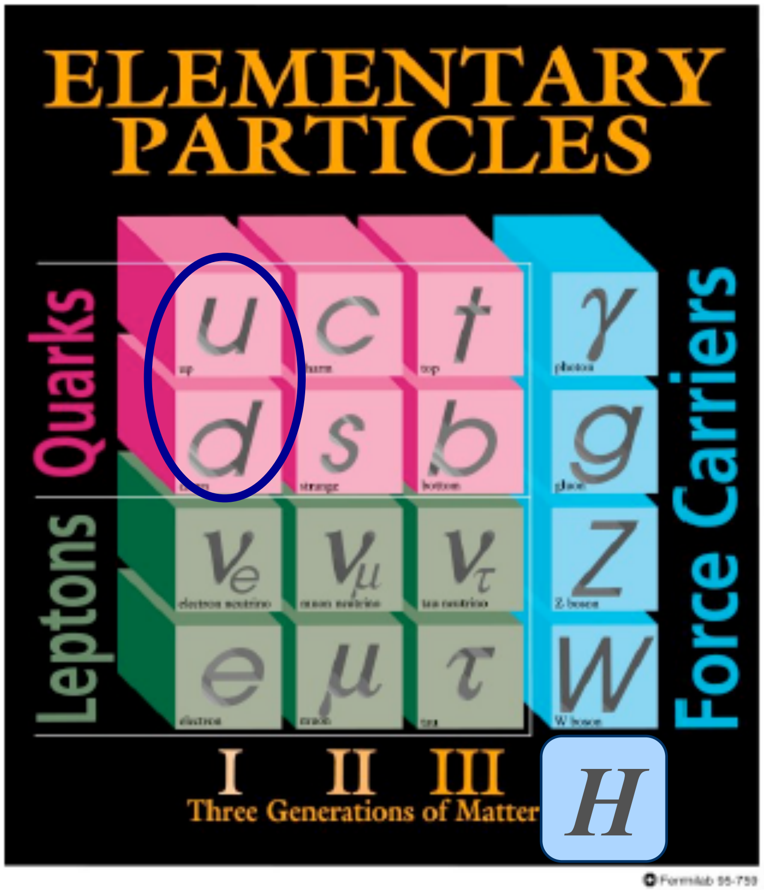
We will come back to the determination of quark mass difference later

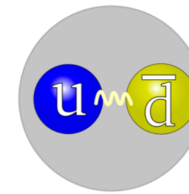
2.5 Success of the Standard Model and search for New Physics

Oscillations of Kaons

- Let us consider simplest hadrons: the mesons. They are quark-anti-quark bound states. They interact with strong, electromagnetic and weak forces

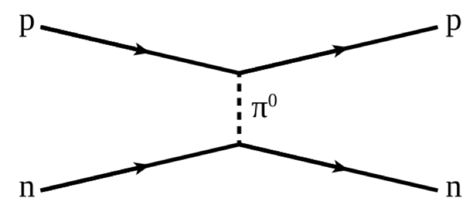


  - The simplest one is the pion:



$$\pi^+ : u\bar{d} \quad , \quad \pi^0 : u\bar{u} \text{ or } d\bar{d}$$

$$\pi^- : \bar{u}d$$



The pions mediate strong force in nuclei
It is ubiquitous in hadronic collisions

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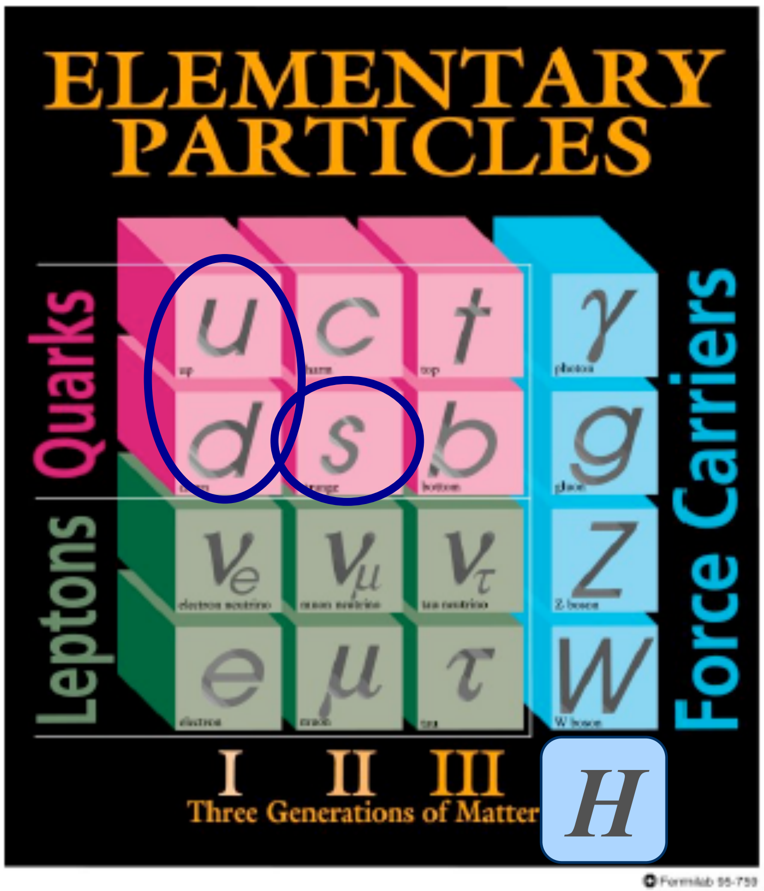

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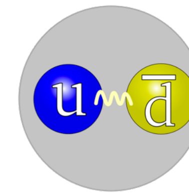

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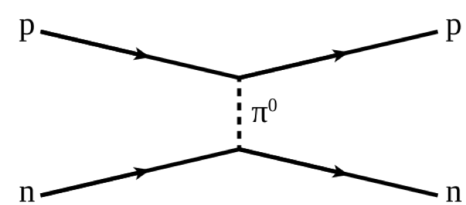
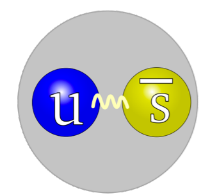


- The ones containing a s quark are the kaons



$$K^+ : u\bar{s}, \quad K^0 : \mathrm{d}\bar{\mathrm{s}}, \bar{K}^0 : s\bar{d}$$

$$K^- : \bar{u}s$$

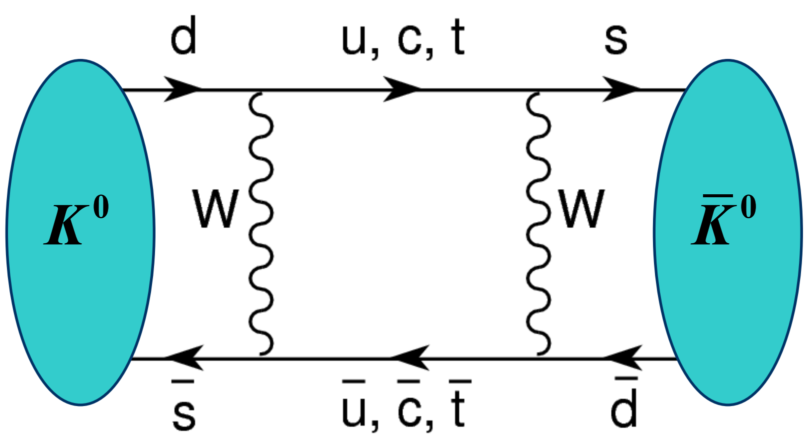
Discovered in *cosmic ray experiments*

Oscillations of Kaons

- Discovered in 1964 by Christenson, Cronin, Fitch and Turlay

  ⇨ Nobel Prize in 1980 for Cronin and Fitch

- Start with a K^0 ⇨ after some time it transforms into a $\bar{K}^0$



through weak interaction
Short distance effect

- The rate of this oscillation is suppressed but measurable in the Standard Model

  ⇨ goes through *weak interactions* $\sim G_F$ $G_F \simeq 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2}$

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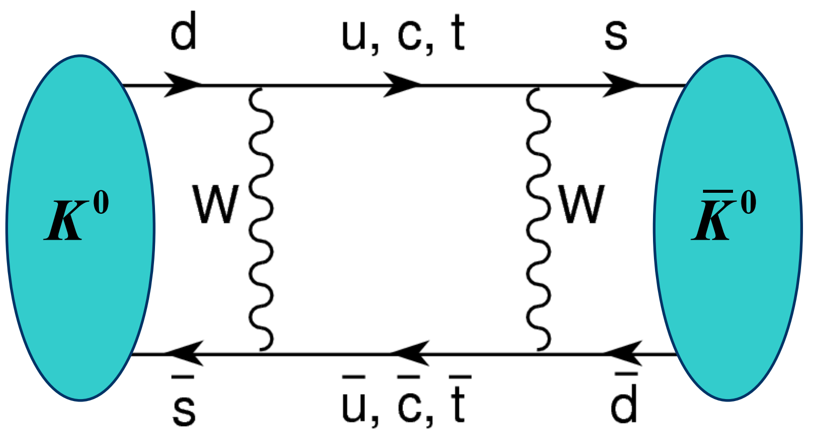


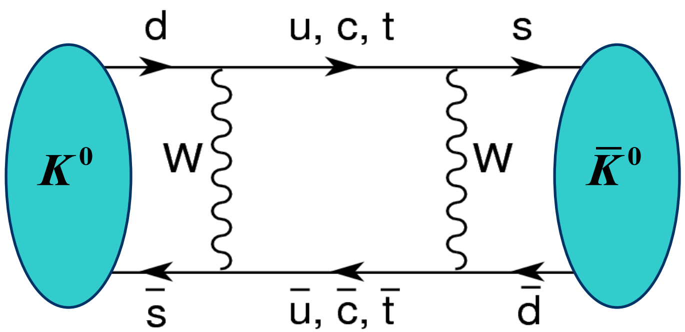
through weak interaction
Short distance effect

- The rate of this oscillation is very suppressed in the Standard Model

  ⇨ goes through *weak interactions* ~ G_F

- How can we understand the oscillation rate?

Oscillations of Kaons



- Process described using the bag parameter B_K
  Fundamental hadronic quantity proportional to matrix element

  ⇨ determined using *lattice QCD*

$$\langle \bar{K}^0 | \mathbf{H} | K^0 \rangle \sim \sum_{ij} \lambda_i \lambda_j \, S(r_i, r_j) \, \eta_{ij} \quad \langle O_{\Delta S=2} \rangle$$

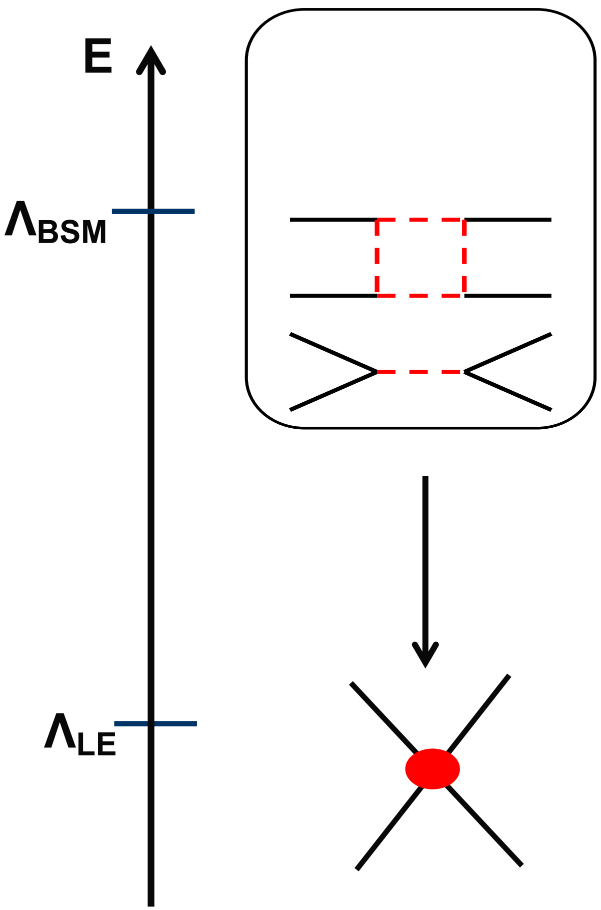
$$\langle O_{\Delta S=2} \rangle = \alpha_s(\mu)^{-2/9} \langle \bar{K}^0 | (\bar{s}_L \gamma^\alpha d_L)(\bar{s}_L \gamma_\alpha d_L) | K^0 \rangle \equiv \left(\frac{4}{3} M_K^2 f_K^2 \right) \hat{B}_K$$

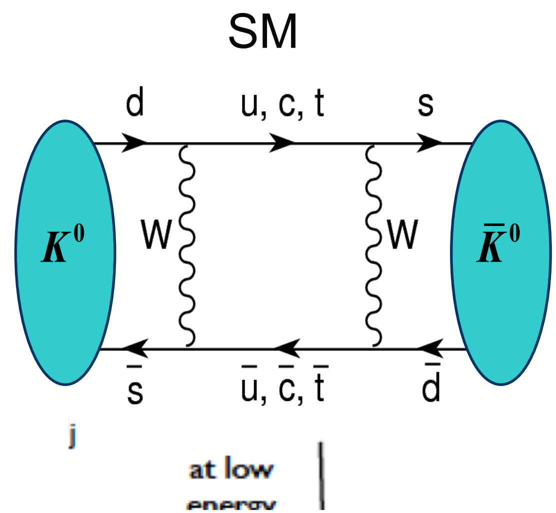
$$\lambda_i \equiv V_{id} V_{is}^* \quad ; \quad r_i \equiv m_i^2 / M_W^2 \quad (i = u, c, t)$$

Oscillations of Kaons

- Since process is suppressed in the Standard Model:

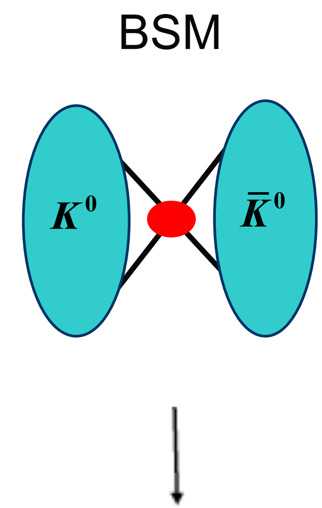
  ⇨ very sensitive to *new physics*: new degrees of freedom and symmetries





$$G_F\ V_{ij}\ V^{\dagger}_{ij}\ \frac{m^2_{i,j}}{M^2_W}$$

+



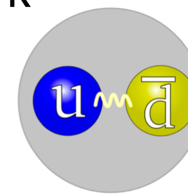
$$1/\Lambda^2$$

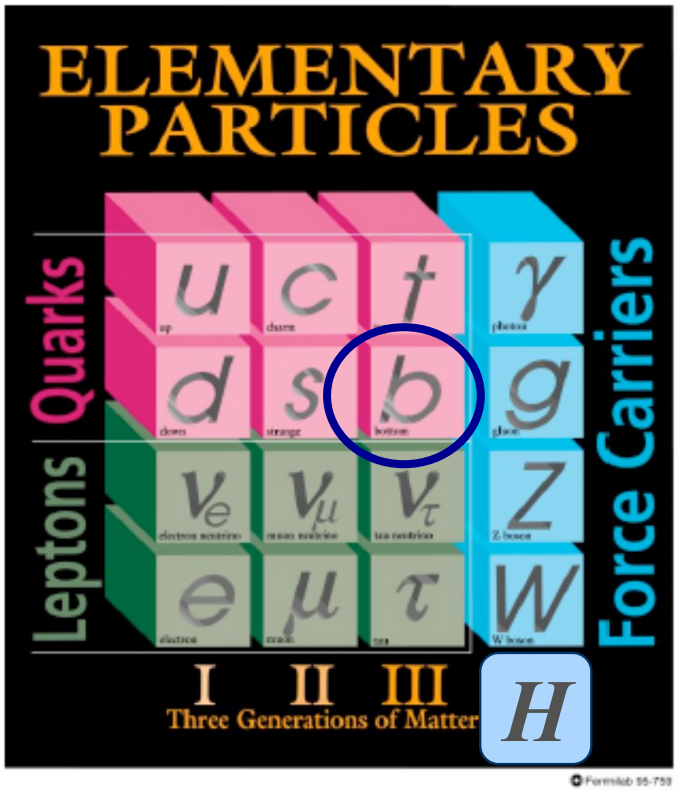
- If measured with very good precision provided the SM contribution is known

  ⇨ *stringent constraints* on new physics models

Oscillations of B mesons

- Similar tests with other mesons ⇒ Beauty mesons contain a b-quark





$B^+ : u\bar{b}$, $B^0 : d\bar{b}$

$B^- : \bar{u}b$, $\bar{B}^0 : \bar{d}b$

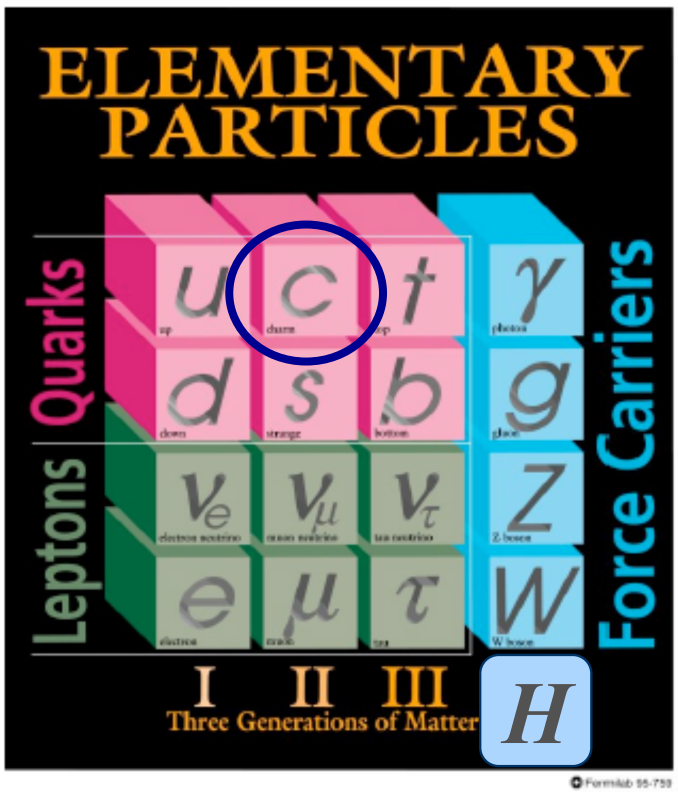
$B_s^0 : s\bar{b}$, $\bar{B}_s^0 : \bar{s}b$

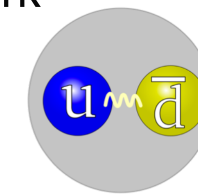
$B_c^0 : c\bar{b}$, $B_c^0 : \bar{c}b$

- B meson physics have been studied extensively at BaBar, Belle, CDF, D0@Tevatron and now Belle-II, LHCb, CMS and ATLAS@LHC

Oscillations of B mesons

- Similar tests with other mesons ⇒ Beauty mesons contain a b-quark





$$B^+ : u\bar{b} \; , \qquad B^0 : d\bar{b}$$

$$B^- : \bar{u}b \; , \qquad \bar{B}^0 : \bar{d}b$$

$$B_s^0 : s\bar{b} \; , \qquad \bar{B}_s^0 : \bar{s}b$$

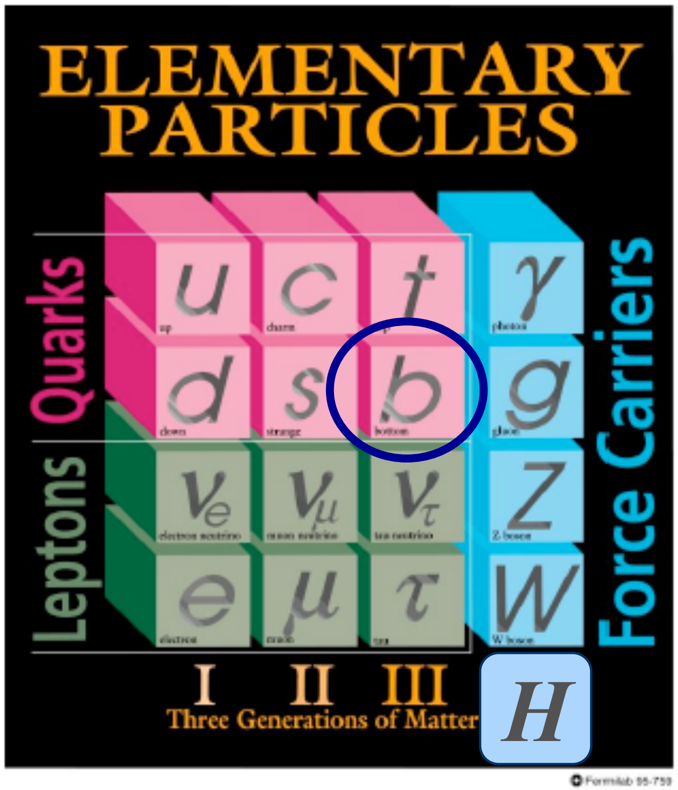
$$B_c^0 : c\bar{b} \; , \qquad B_c^0 : \bar{c}b$$

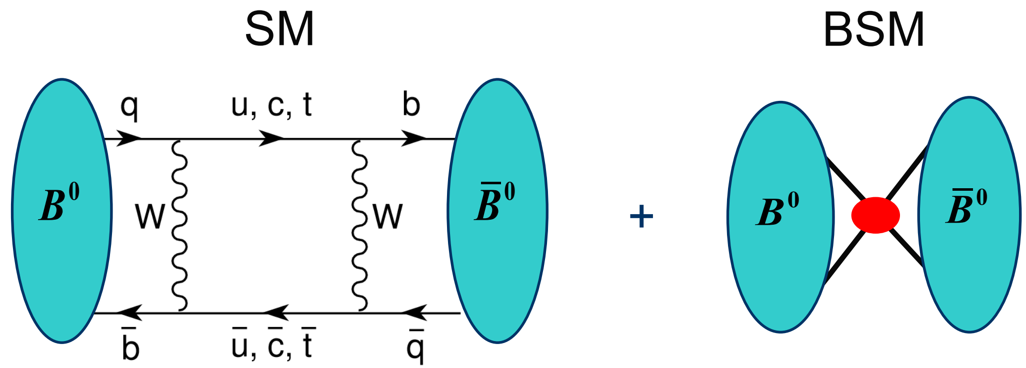
- B meson physics have been studied extensively at BaBar, Belle, CDF, D0@Tevatron and now Belle-II, LHCb, CMS and ATLAS@LHC

- Similar tests with D mesons

Oscillations of B mesons

- Similar tests with other mesons





- B-Bbar measured by *BaBar* and *Belle'01*

- Bs-Bsbar mixing observed by *CDF'06* and *LHCb'11*

CP violation in B decays *LHCb'13*

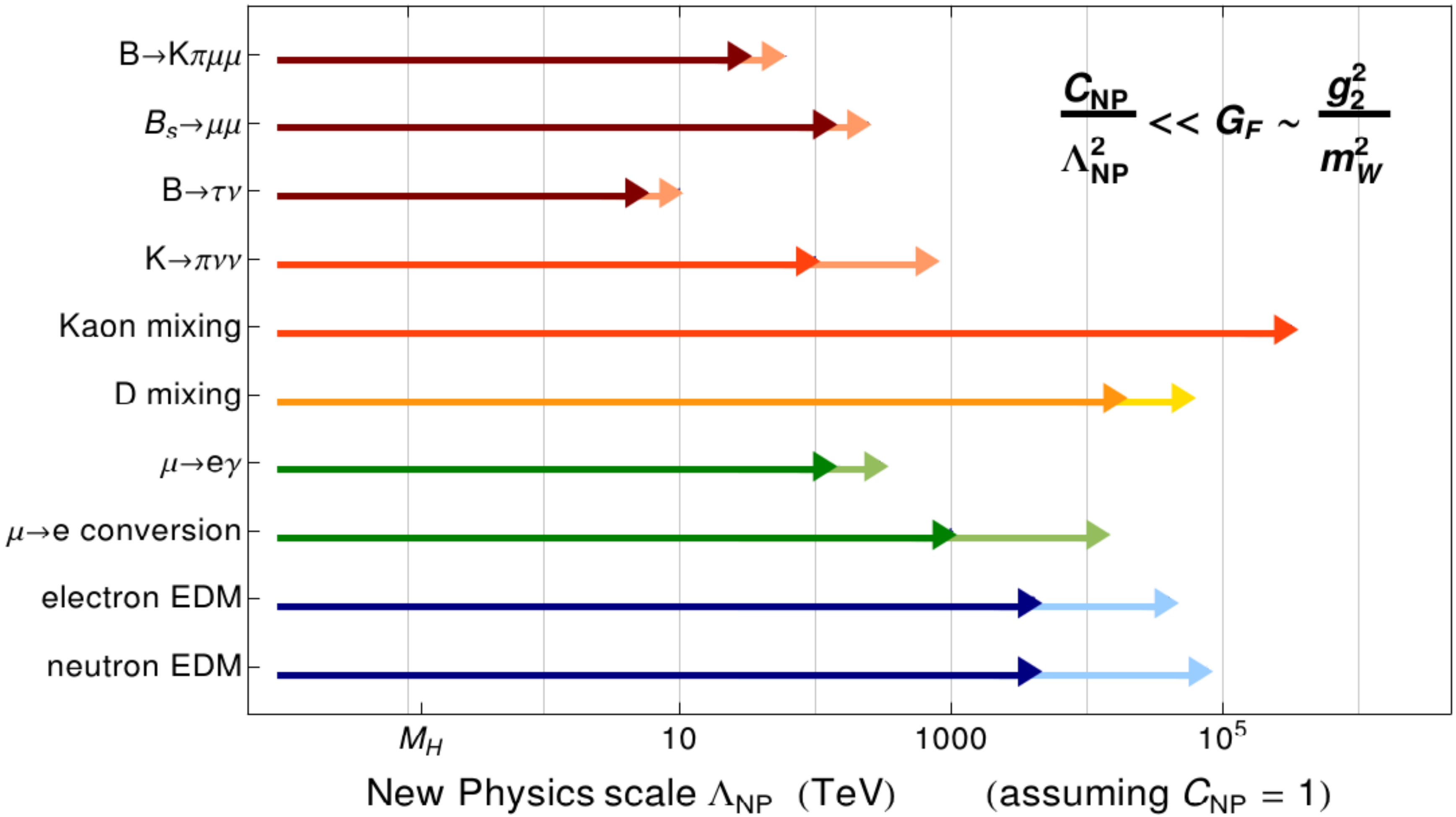
CP violation in D decays *LHCb'19*

- Stringent constraints on new physics models provided *hadronic* matrix elements known

New Physics and Flavour sector

- Very sensitive to New Physics

W. Altmannshofer



Anomalies in Flavour Physics

- Exciting discrepancies found recently:

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2 Accelerators Find Particles That May Break Known Laws of Physics

The LHC and the Belle experiment have found particle decay patterns that violate the Standard Model of particle physics, confirming earlier observations at the BaBar facility

By Clara Moskowitz | September 9, 2015 | Véalo en español

physicstoday

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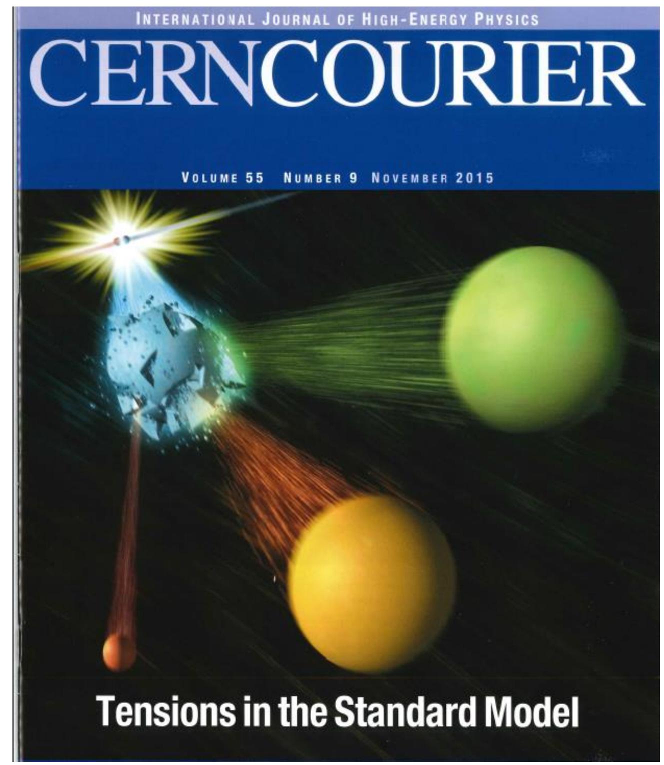
Democracy suffers a blow—in particle physics

Three independent B-meson experiments suggest that the charged leptons may not be so equal after all.

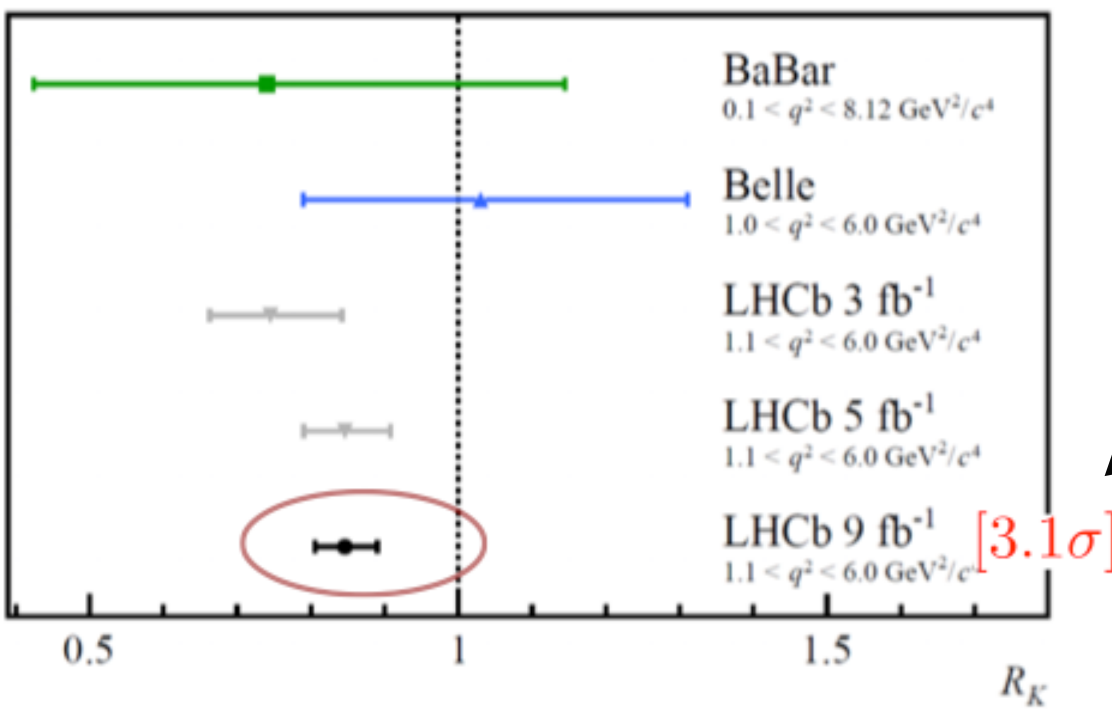
Steven K. Blau 17 September 2015



Anomalies in Flavour Physics

- Exciting discrepancies found recently:

$$R_{K^{(*)}} = \left.\frac{\mathcal{B}(B \to K^{(*)}\mu\mu)}{\mathcal{B}(B \to K^{(*)}ee)}\right|_{q^2 \in [q^2_{\rm min}, q^2_{\rm max}]}$$

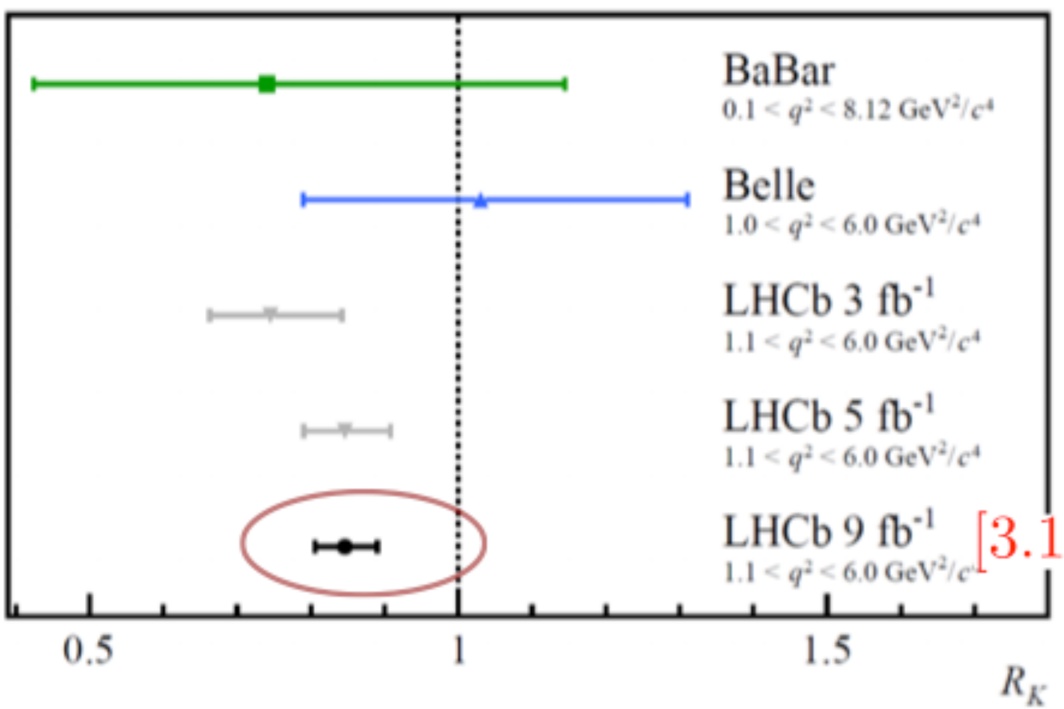


- Hadronic uncertainties cancel in the ratio
- Update from *LHCb @Moriond 2021*

Anomalies in Flavour Physics

- Exciting discrepancies found recently:

$$R_{K^{(*)}} = \frac{\mathcal{B}(B \to K^{(*)}\mu\mu)}{\mathcal{B}(B \to K^{(*)}ee)}\Bigg|_{q^2\in[q^2_{\rm min},q^2_{\rm max}]}$$



$[3.1\sigma]$

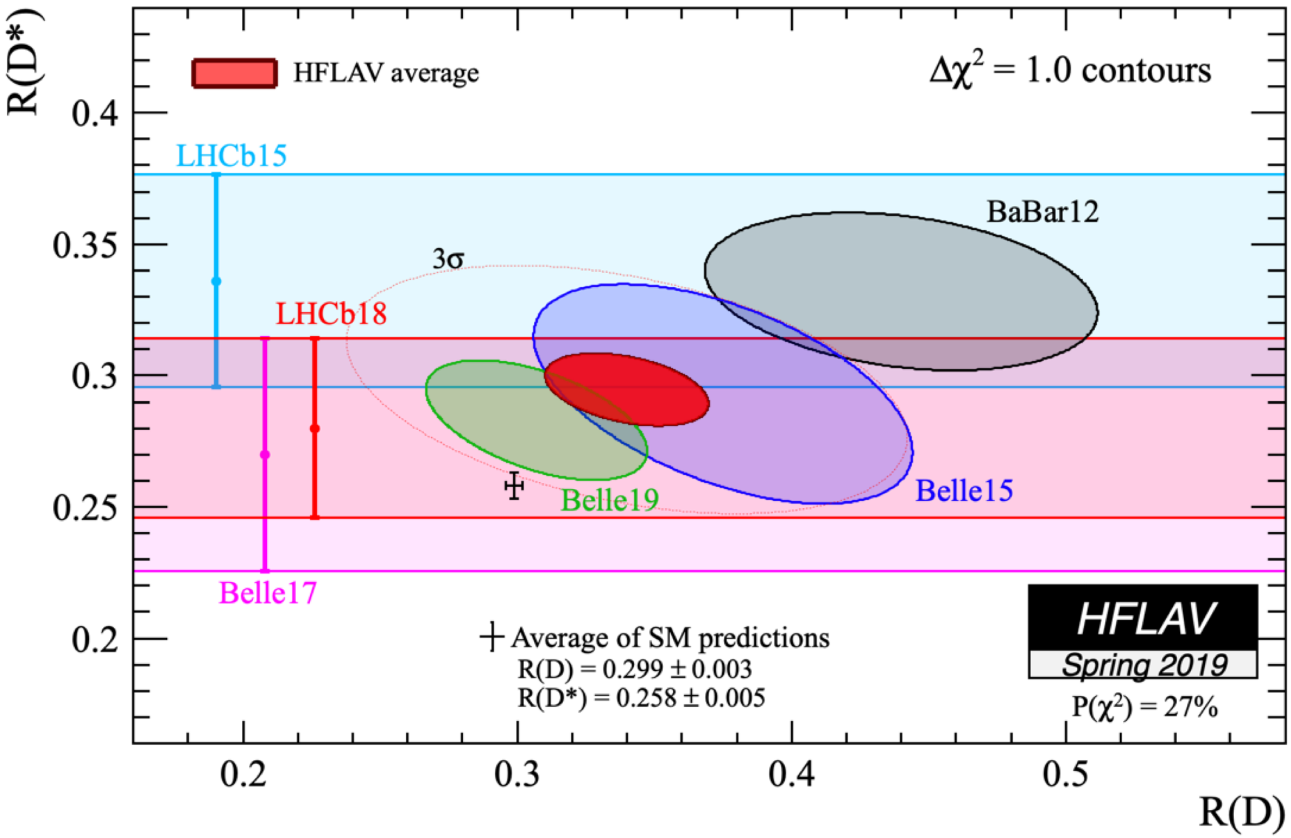
- Hadronic uncertainties cancel in the ratio
- Update from *LHCb @Moriond 2021*

$$\& \quad R^{\rm exp}_{K^{(*)}} < R^{\rm SM}_{K^{(*)}}$$

Anomalies in Flavour Physics

- Exciting discrepancies found recently:

$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)} \tau \bar{\nu})}{\mathcal{B}(B \to D^{(*)} \ell \bar{\nu})}_{\ell \in (e,\mu)}$$



- Hadronic uncertainties cancel in the ratio: The SM prediction very precise

Anomalies in Flavour Physics

- These anomalies have generated a lot of excitement and theoretical papers to try to explain them using new physics models

- This requires a good understanding of hadronic physics
  see e.g. *Celis, Cirigliano, E.P., Phys.Rev. D89 (2014) 013008,*
  Phys.Rev. D89 (2014) no.9, 095014

- New measurements are planned at ATLAS, CMS (dedicated B physics run) LHCb and Belle II

- Better precision within the next decade ⇨ match the level of precision theoretically with *hadronic physics*

3. Selected examples: $\eta \rightarrow 3\pi$ and light quark mass ratio

Colangelo, Lanz, Leutwyler, E. P., PRL 118 (2017) no.2, 022001, EPJC78 (2018) no.11, 947

Review on η and η’ physics: *Gan, Kubis, E.P., Tulin, ArXiv: 2007.00664[hep-ph]*

Introduction

PDG'21

- η decay from PDG:

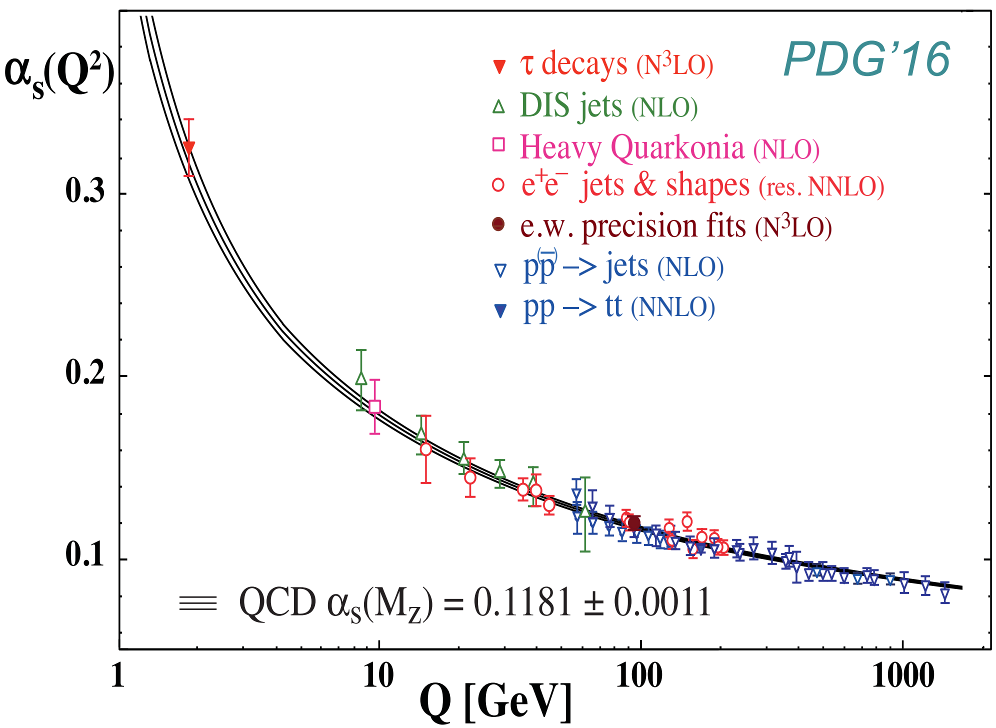
η DECAY MODES

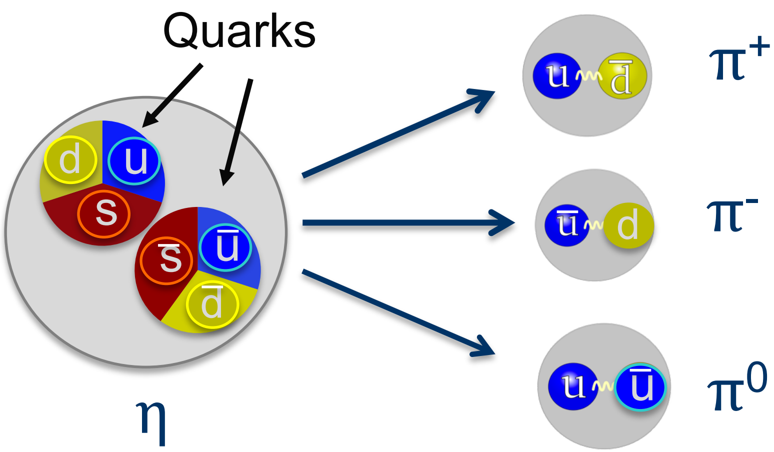
	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
	Neutral modes		
Γ_1	neutral modes	(72.12±0.34) %	S=1.2
Γ_2	2γ	(39.41±0.20) %	S=1.1
Γ_3	$3\pi^0$	(32.68±0.23) %	S=1.1
	Charged modes		
Γ_8	charged modes	(28.10±0.34) %	S=1.2
Γ_9	$\pi^+\pi^-\pi^0$	(22.92±0.28) %	S=1.2
Γ_{10}	$\pi^+\pi^-\gamma$	(4.22±0.08) %	S=1.1

Introduction

- $\eta \rightarrow 3\pi$ forbidden by isospin symmetry ⇨ Unique access to $(m_d - m_u)$

- $M_\eta = 547.862(117)$ **MeV**





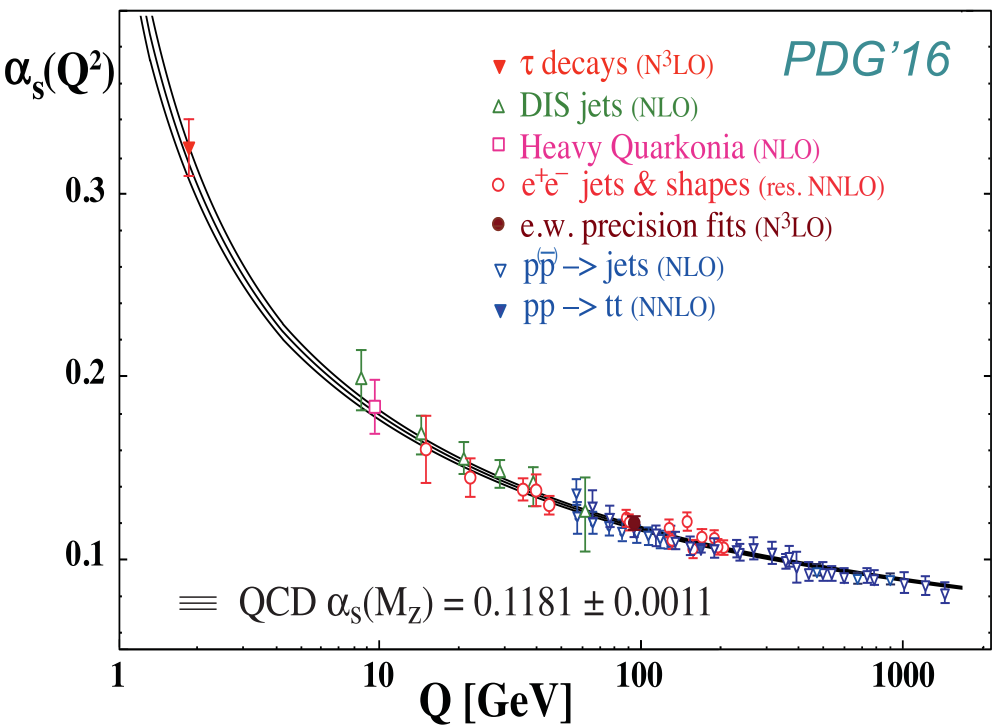
⇨ Non perturbative methods

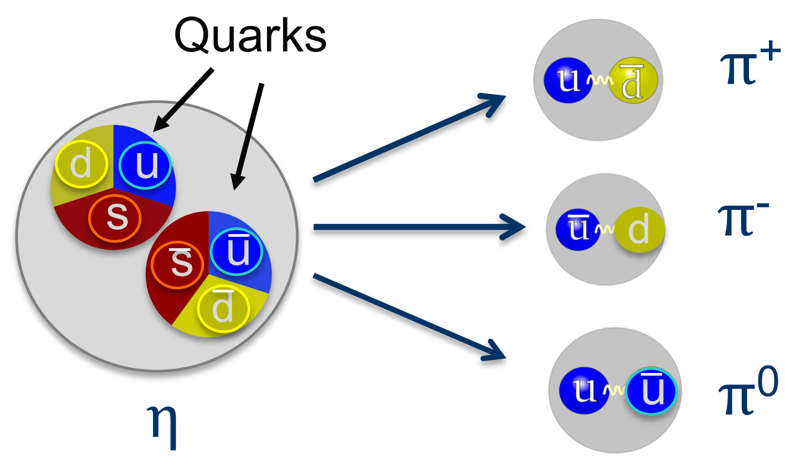
Introduction

- $\eta \to 3\pi$ forbidden by isospin symmetry ⇨ Unique access to ($m_d - m_u$)

- $M_\eta = 547.862(117)$ MeV

  Too low for perturbative QCD





⇨ *Non perturbative methods*

$$\langle \pi^+\pi^-\pi^0_{out} | \eta \rangle = i(2\pi)^4 \delta^4 \left(p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0}\right) A(s,t,u)$$

Chiral Perturbation Theory

- *Ch*iral *P*erturbation *T*heory (*ChPT*): Effective field theory in the light quark sector
- Hadronic energy scale ($\Lambda_H \sim 1$ GeV) ⇨ Light mesons and their interaction
  - Degrees of freedom: light mesons (Goldstone Bosons): π, K, η
  - Chiral symmetry
- New parameter of expansion $\frac{\alpha_S(\mu)}{\pi}$ ⇨ $\frac{p}{\Lambda_H}$ + small light quark masses

⇨ $$\sigma = \sigma_0 + \left(\frac{p}{\Lambda_H}\right)^2 \sigma_2 + \left(\frac{p}{\Lambda_H}\right)^4 \sigma_4 + \ldots$$

- Validity: $\mathrm{p} \ll \Lambda_H \sim 1\ \mathrm{GeV}$

$\eta \to 3\pi$ in ChPT

- Compute the amplitude using ChPT :

$$\Gamma_{\eta\to 3\pi} = (66 + 94 + ... + ...)\,\text{eV} = (300 \pm 12)\,\text{eV}$$

LO NLO NNLO

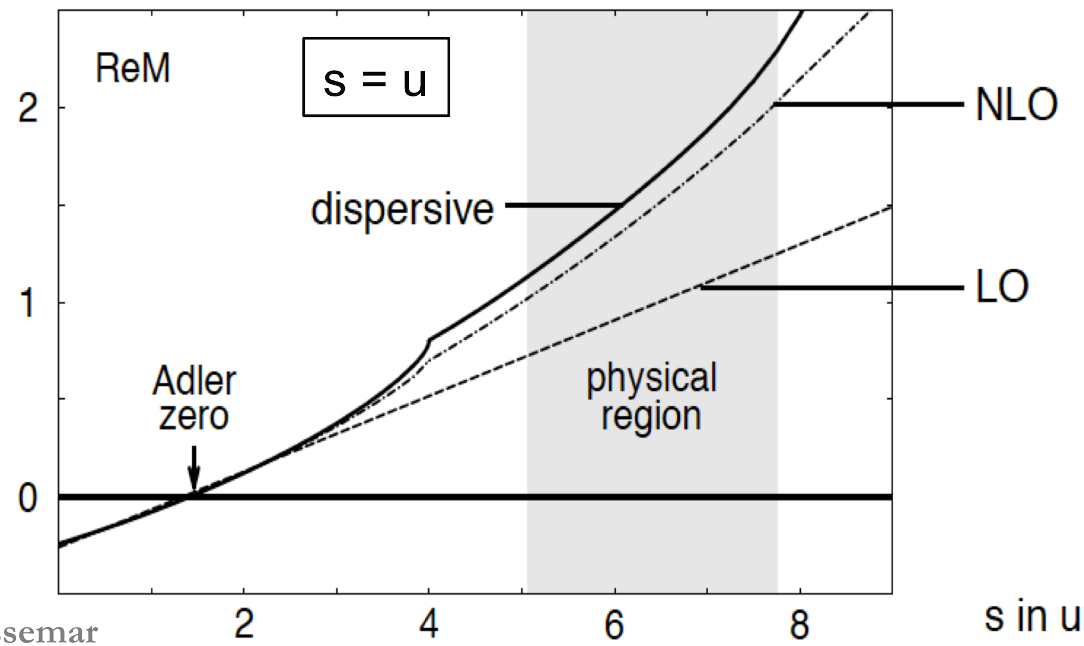
PDG'16

LO: *Osborn, Wallace'70*

NLO: *Gasser & Leutwyler'85*

NNLO: *Bijnens & Ghorbani'07*

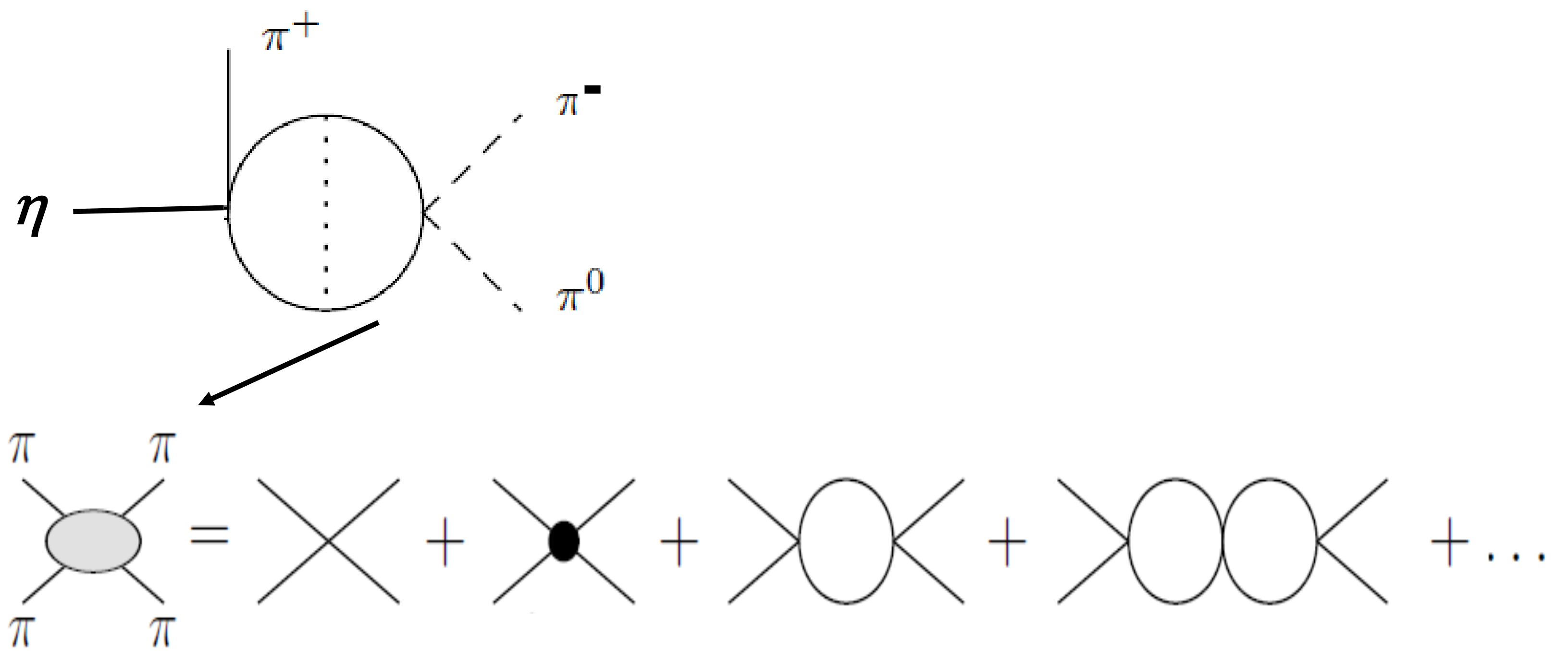
- The Chiral series has convergence problems



Dispersive approach

- The Chiral series has convergence problems

  ➡ Large *ππ final state interactions* *Roiesnel & Truong'81*



- *Dispersive treatment :*
  – analyticity, unitarity and crossing symmetry
  – Take into account all the rescattering effects

Why a new dispersive analysis?

- Several new ingredients:
  - New inputs available: extraction $\pi\pi$ phase shifts has improved

    Garcia-Martin et al'09, Colangelo et al.'11

  - New experimental programs, precise Dalitz plot measurements

    TAPS/CBall-MAMI (Mainz), WASA-Celsius (Uppsala), WASA-Cosy (Juelich)
    CBall-Brookhaven, CLAS, GlueX (JLab), KLOE I-II (Frascati)
    BES III (Beijing)

  - Many improvements needed in view of very precise data: inclusion of
    - Electromagnetic effects ($\mathcal{O}(e^2m)$) *Ditsche, Kubis, Meissner'09*
    - Isospin breaking effects

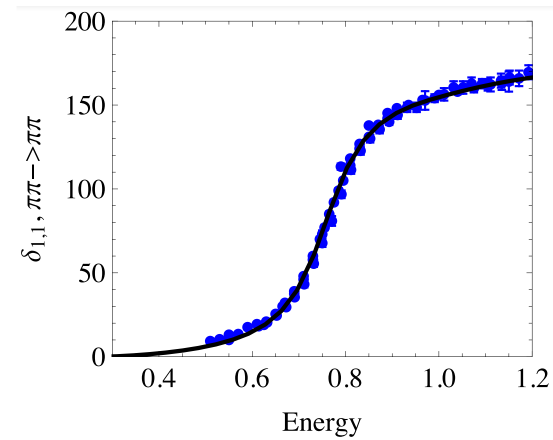
Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$M(s,t,u) = M_0^0(s) + (s-u)M_1^1(t) + (s-t)M_1^1(u) + M_0^2(t) + M_0^2(u) - \frac{2}{3}M_0^2(s)$$

Roy analysis
Colangelo et al.'11

input

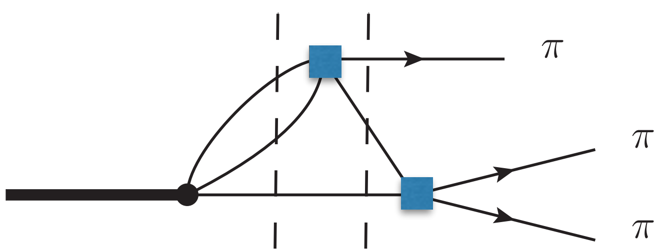


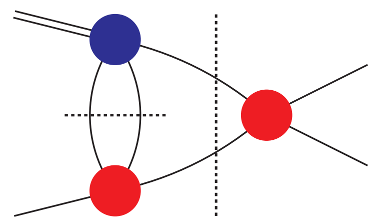
- Unitarity relation:

$\pi\pi \to \pi\pi$

$$disc\left[M_\ell^I(s)\right] = \rho(s)\, t_\ell^*(s)\left(M_\ell^I(s) + \hat{M}_\ell^I(s)\right)$$

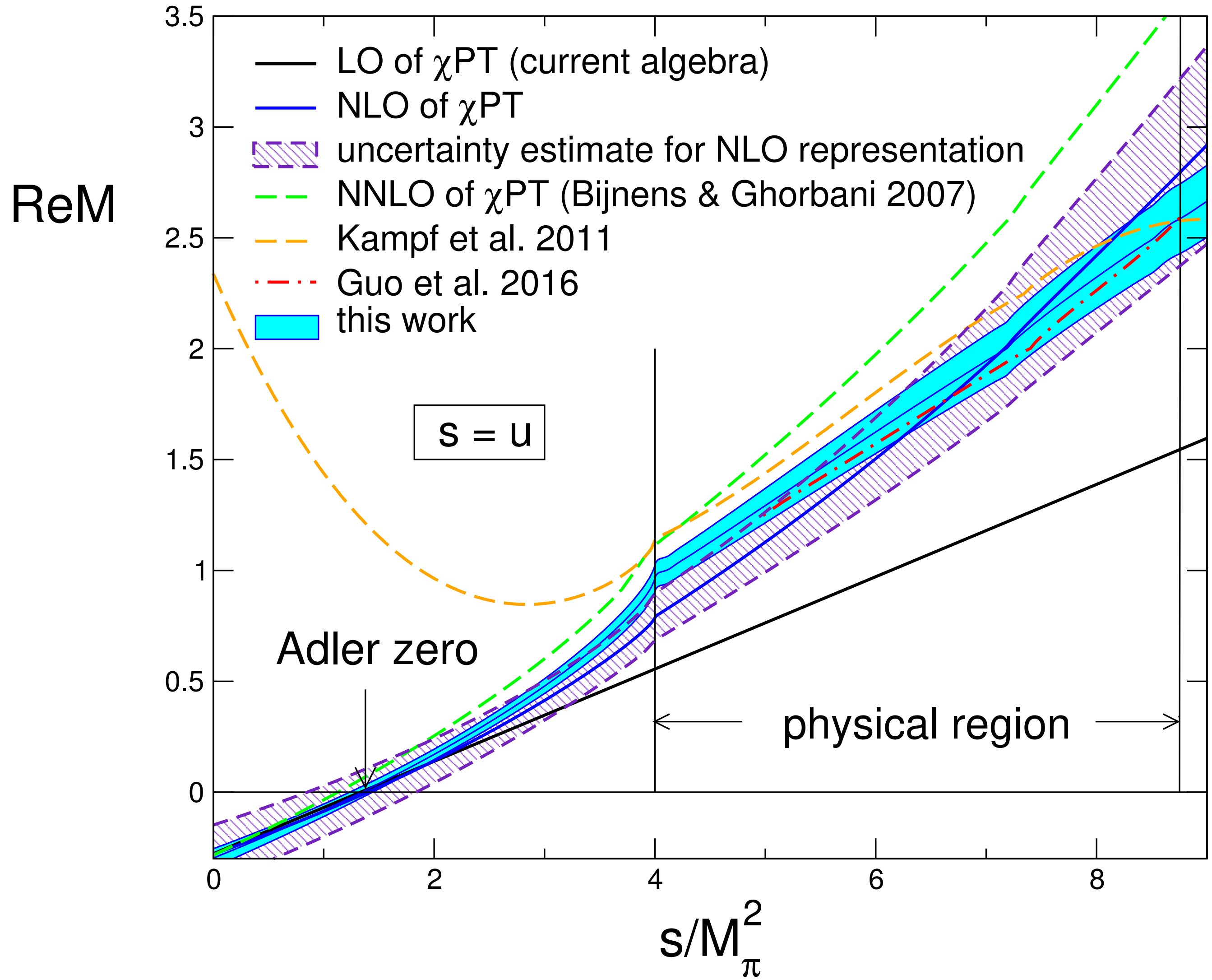
right-hand cut　　left-hand cut





Results: Amplitude for $\eta \to \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line s = u :



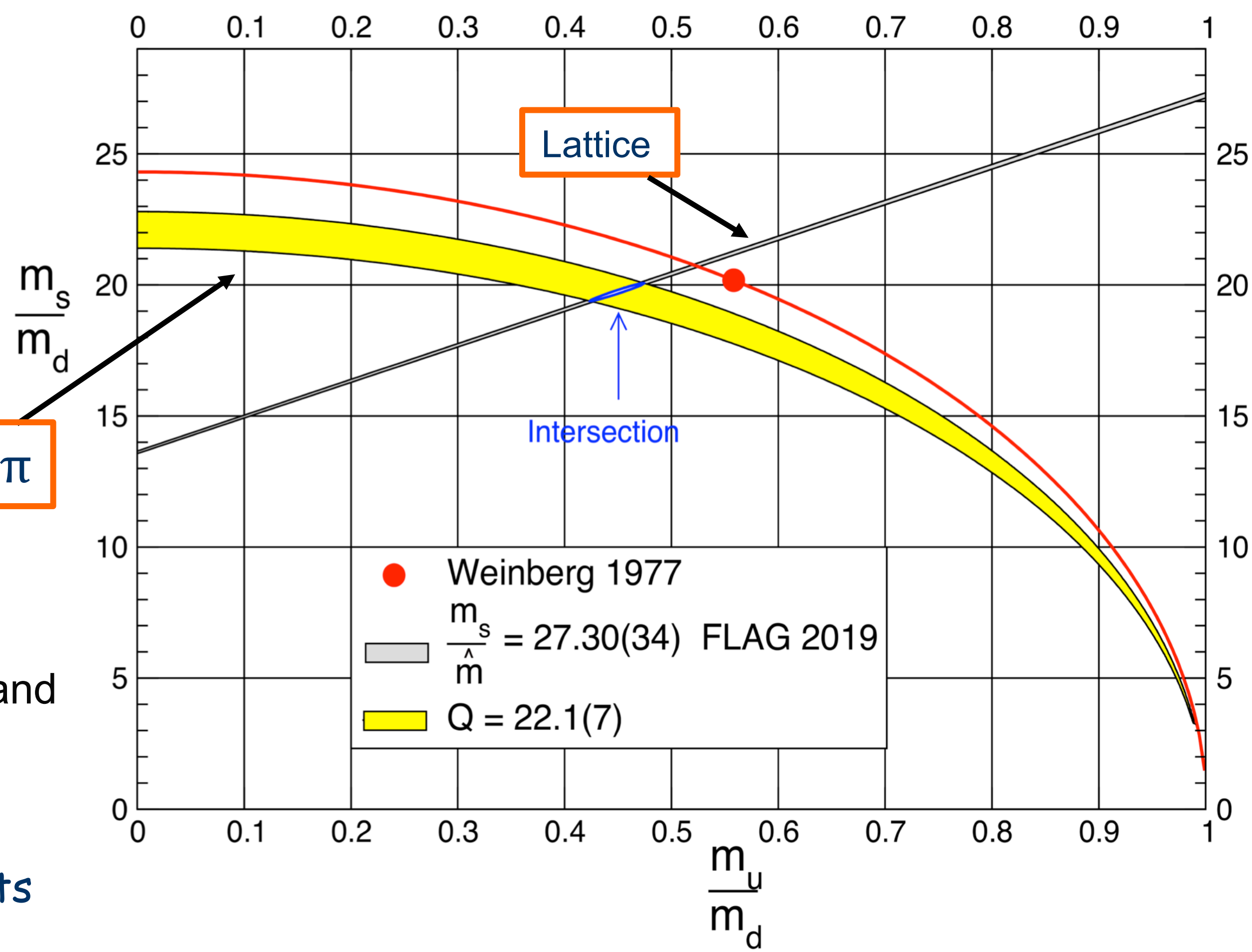
Light quark mass ratio extraction

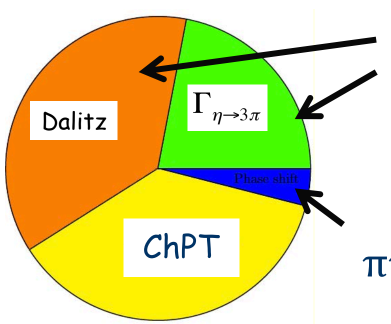
- Extract the light quark mass ratio very precisely
  ⇨ complementary to lattice determination

Shift towards *lower values* of quark mass ratios compared to Current Algebra result

⇨ $$\frac{m_u}{m_d} = 0.44 \pm 0.03$$

$\eta \rightarrow 3\pi$

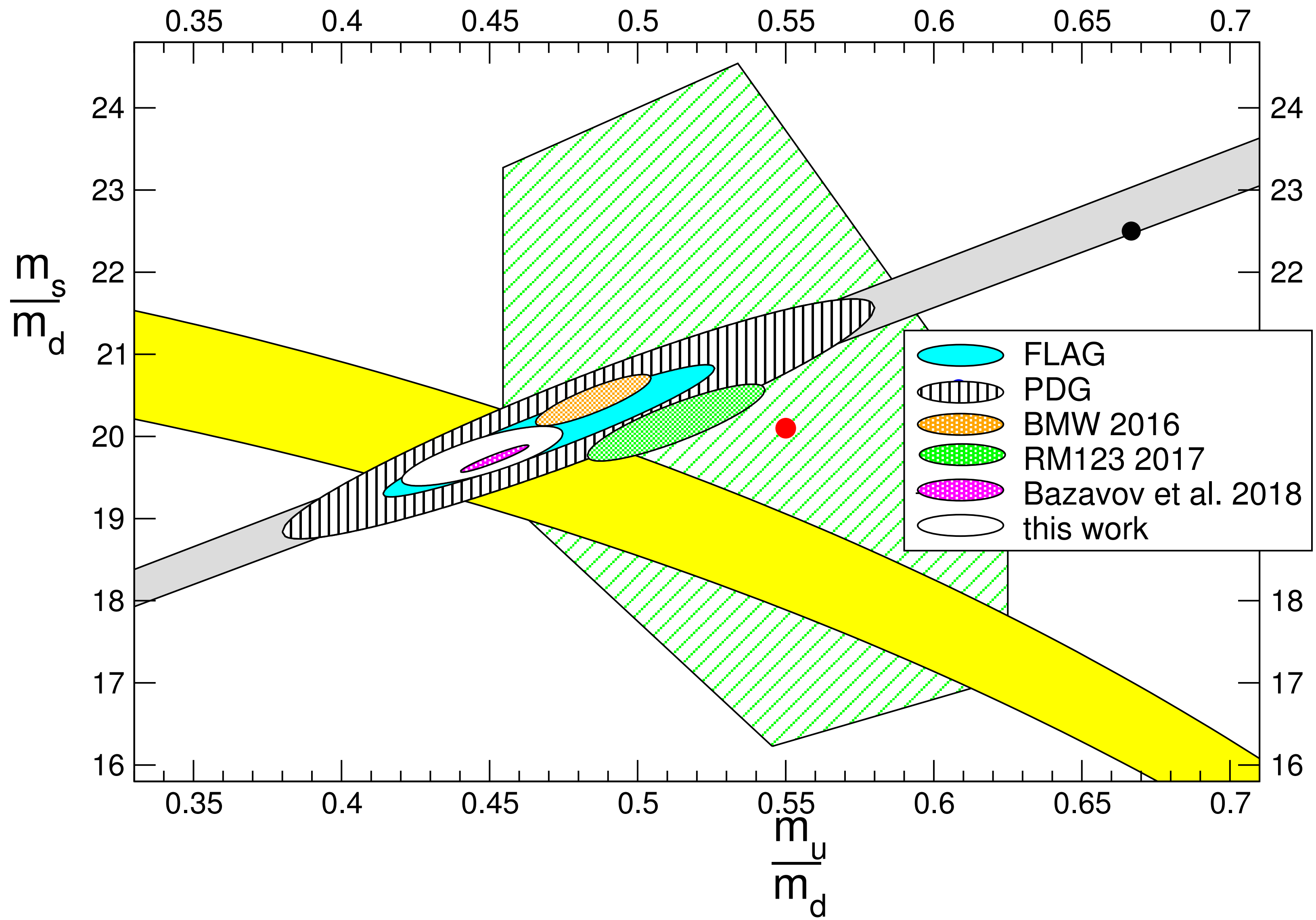




Can be investigated and reduced at *GlueX, JEF, CLAS12*

ππ phase shifts

Light quark mass ratio extraction

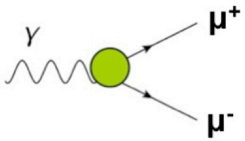


3.2 Anomalous magnetic moment of the muon

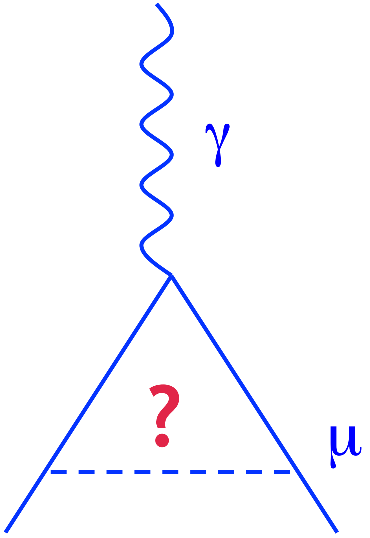
Introduction

$$a_\mu = \frac{(g-2)_\mu}{2}$$

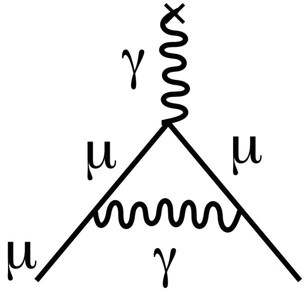
Anomalous magnetic moment

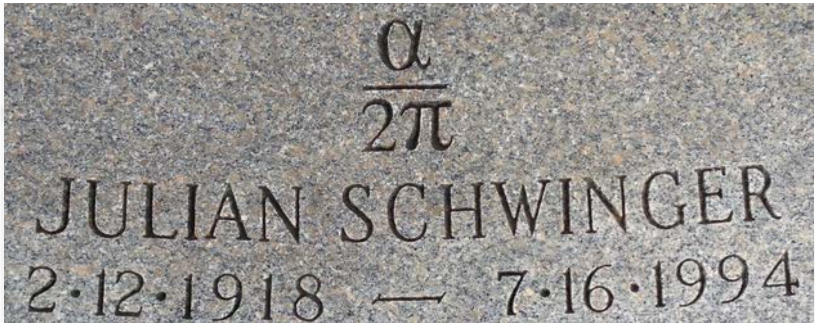


- The gyromagnetic factor of the muon is modified by loop contribution
- Predicted by Dirac to be 2
- Schwinger computed the first order correction



QED

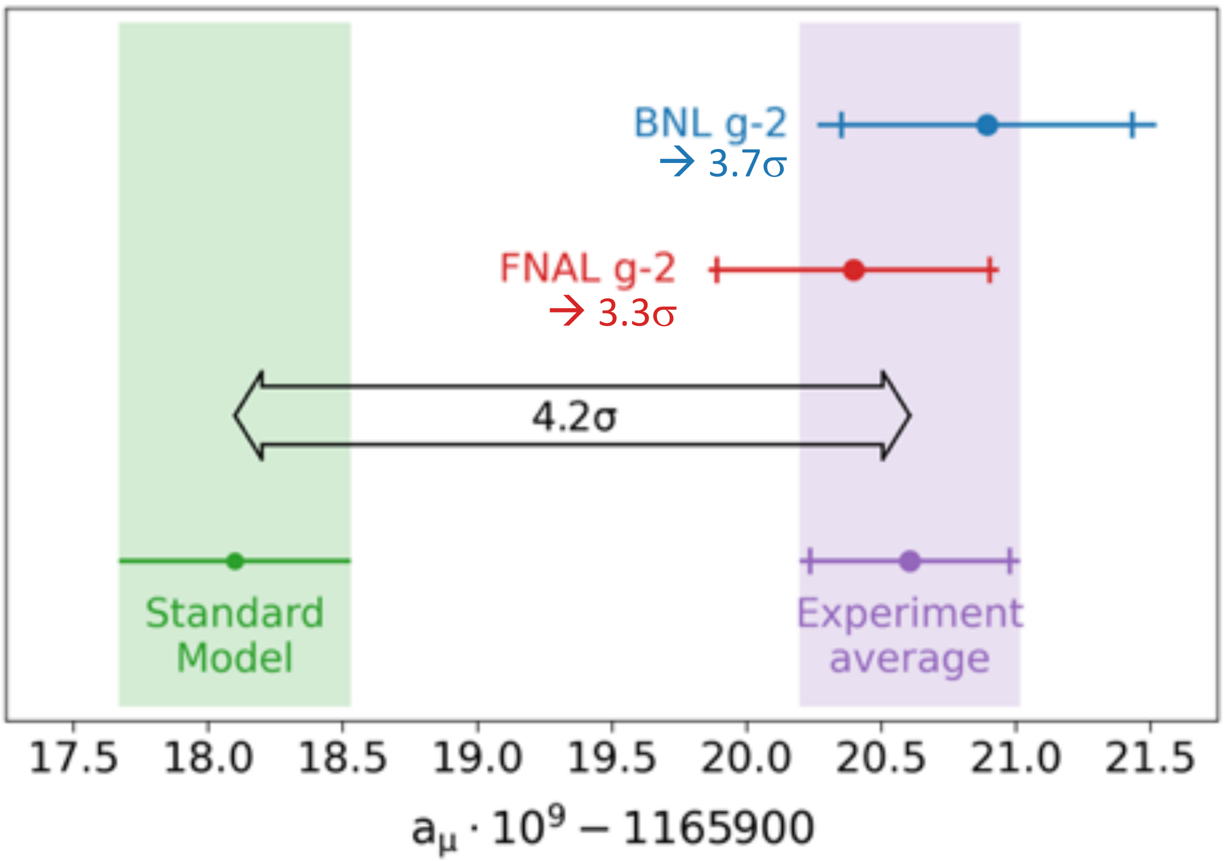




Experimental result

FNAL g-2'21
Chris Polly

$a_\mu(SM) = 0.00116591810(43) \rightarrow 368$ ppb



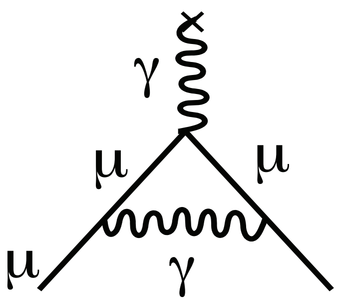
- Individual tension with SM
  - BNL: 3.7σ
  - FNAL: 3.3σ

$a_\mu(Exp) - a_\mu(SM) = 0.00000000251(59) \rightarrow 4.2\sigma$

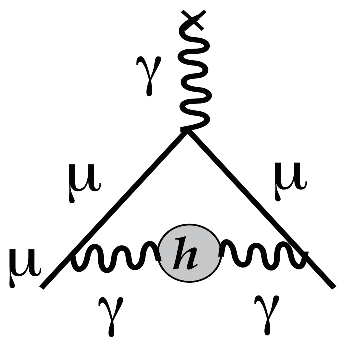
What is the SM prediction?

- Not very easy to obtain at this level of precision

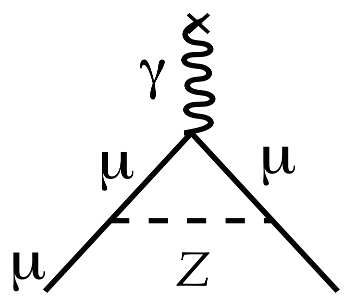
QED



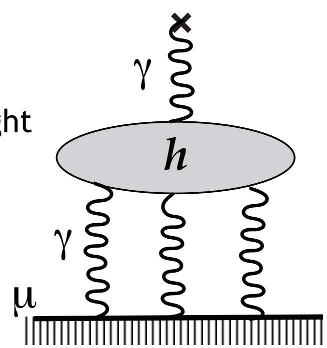
Hadronic

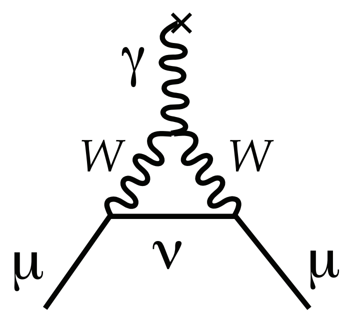


Weak



"Light-by-light scattering"





Muon "(g- 2) Theory Initiative" led by *A. El-Khadra* and *C. Lehner*
White Paper: *Phys.Rept. 887 (2020) 1-166,*
ArXiv: 2006.04822 [hep-ph]

What is the SM prediction?

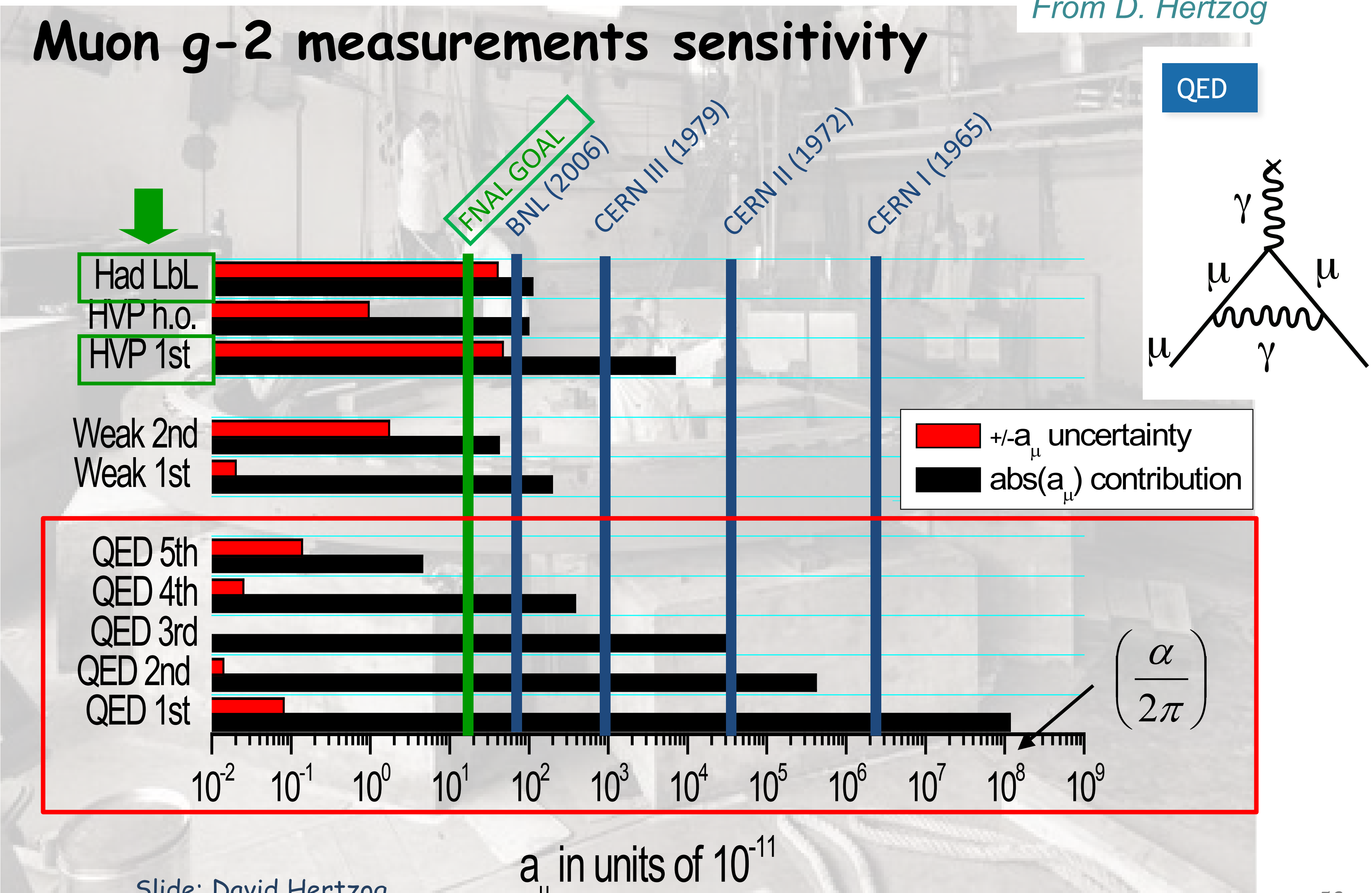
- Theoretical Prediction:

g-2 Theory Initiative White Paper'20

Contribution	Result in 10^{-10} units
QED(leptons)	11658471.893 ± 0.010
HVP(leading order)	693.1 ± 4.0
HVP(higher order)	-8.59 ± 0.71
HLBL	9.2 ± 1.8
EW	15.4 ± 0.1
Total	11659181.0 ± 4.3

On the importance of hadronic contributions

From D. Hertzog



On the importance of hadronic contributions

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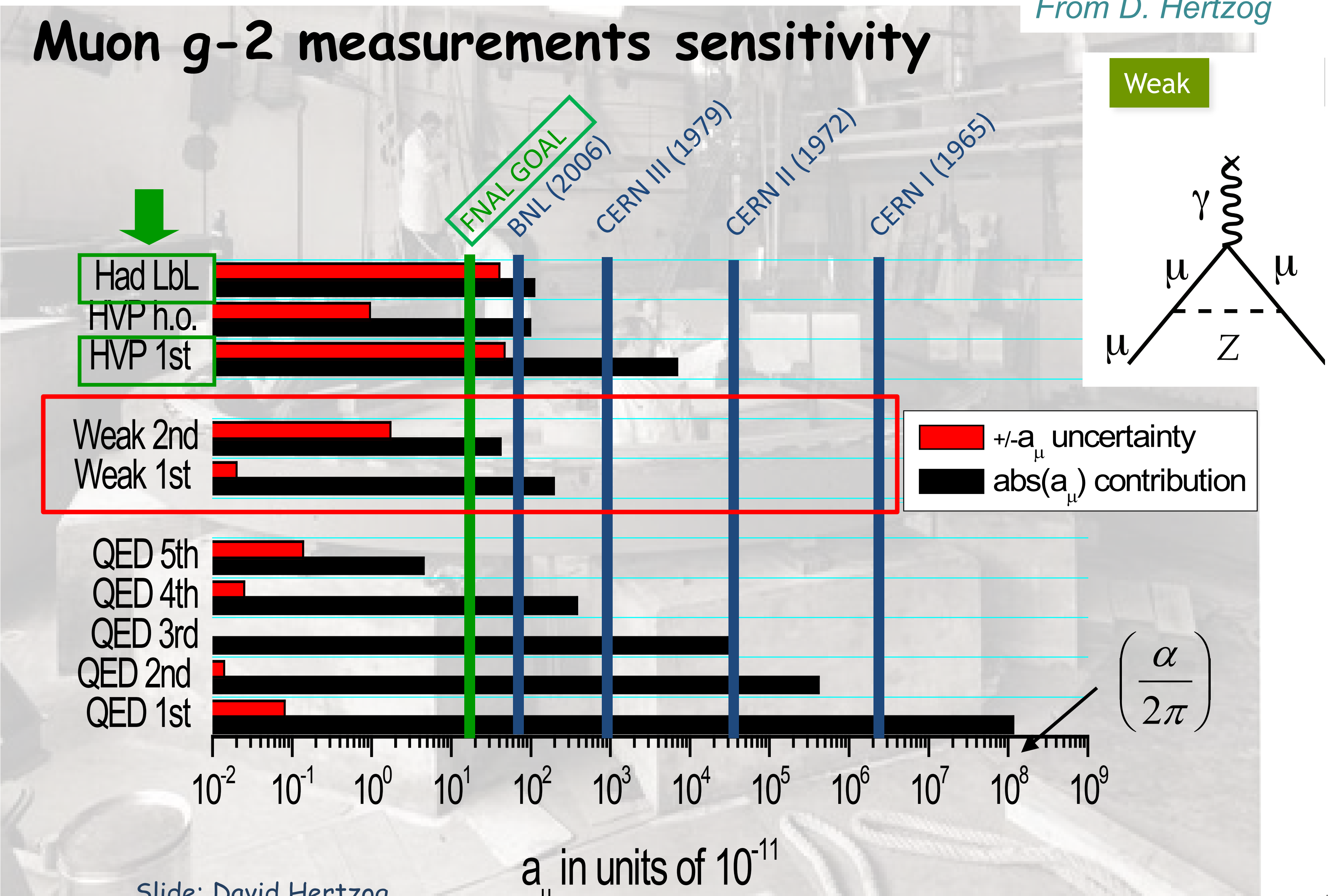
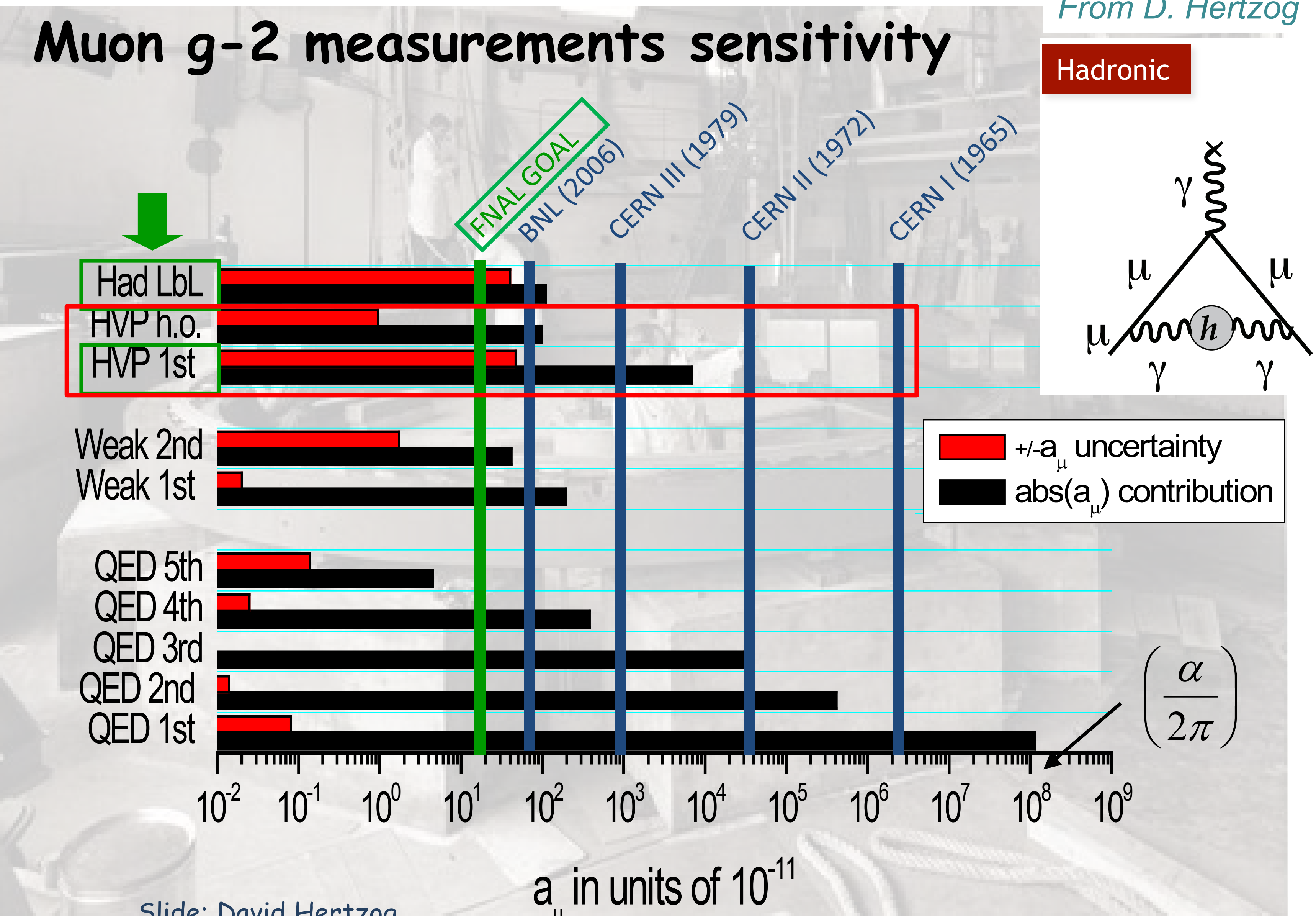


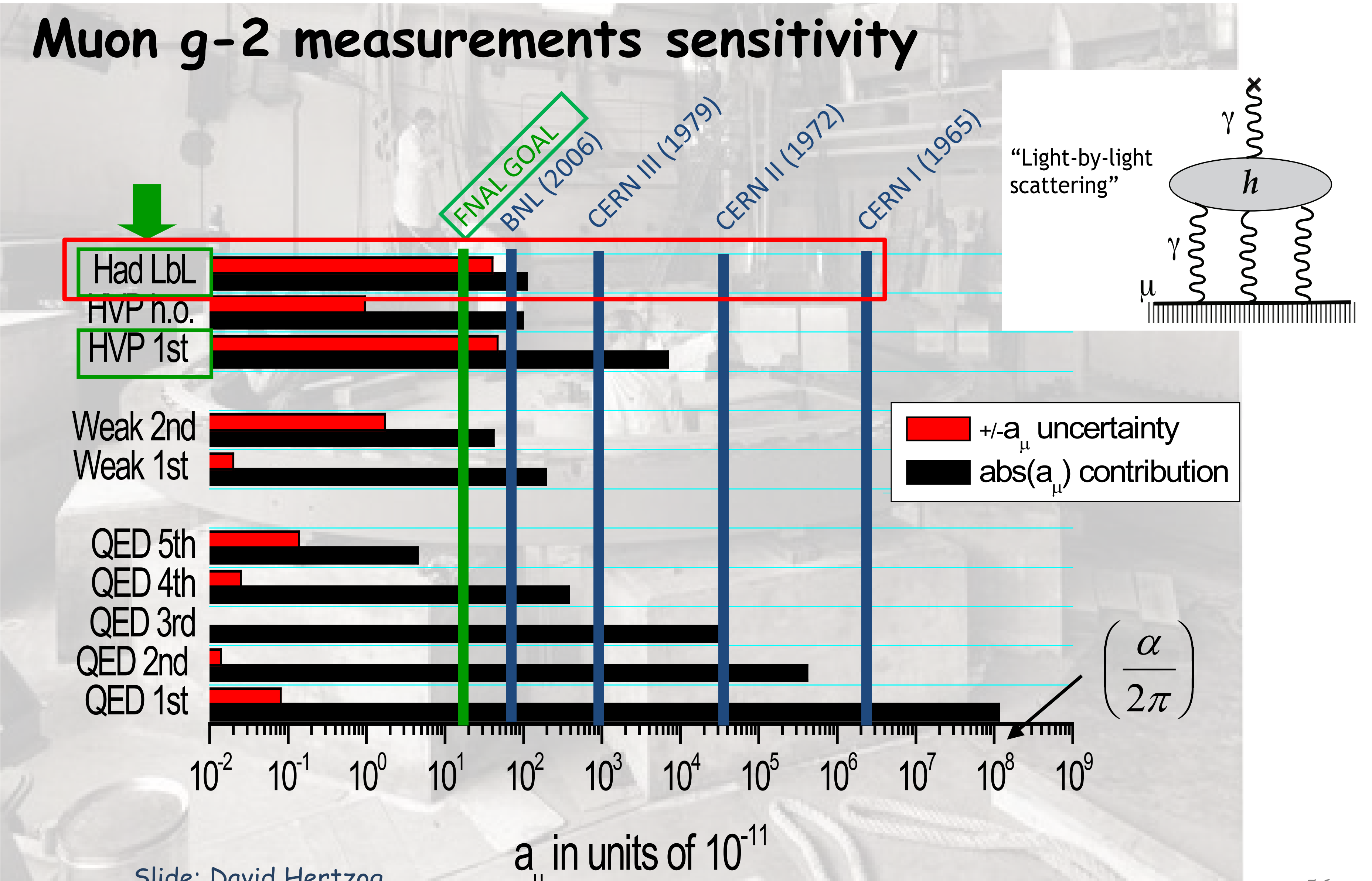
Slide: David Hertzog

On the importance of hadronic contributions

From D. Hertzog



On the importance of hadronic contributions



Slide: David Hertzog

On the importance of hadronic contributions

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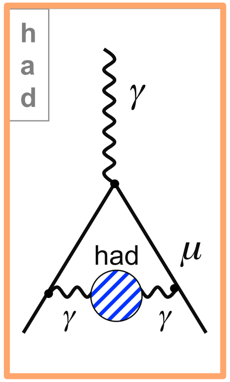
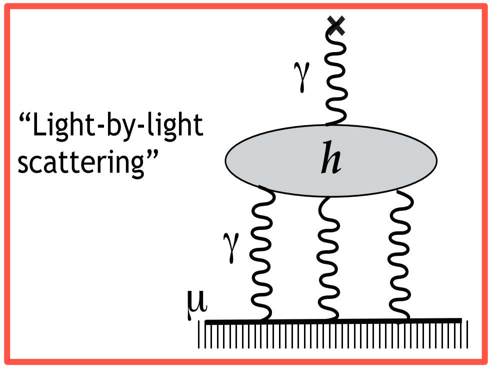
Total	11659181.0 ± 4.3

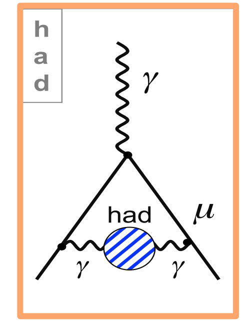
- Important contribution comes from *virtual hadrons* in the loop!

- Tackled using :
  - Models
  - Dispersion Relations
  - Lattice QCD



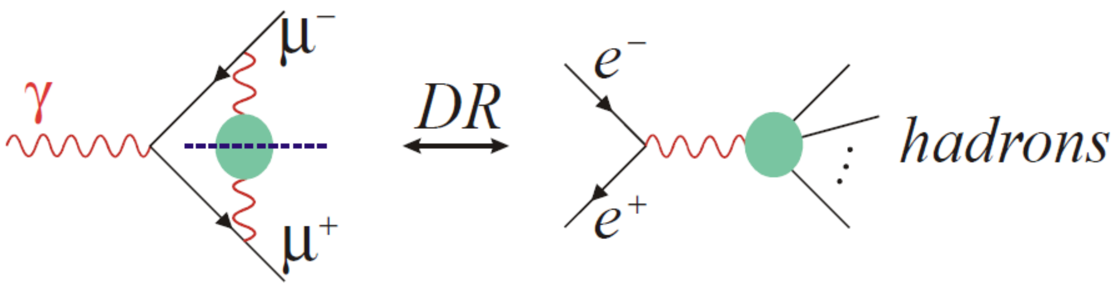


Computation of HVP using data



- Hadronic contribution cannot be computed from first principles due to low-energy hadronic effects

- Use analyticity + unitarity ⇒ real part of photon polarisation function from *dispersion relation* over *total hadronic cross section data*



$$R_V(s) = \frac{\sigma\left(e^+e^- \to hadrons\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)}$$

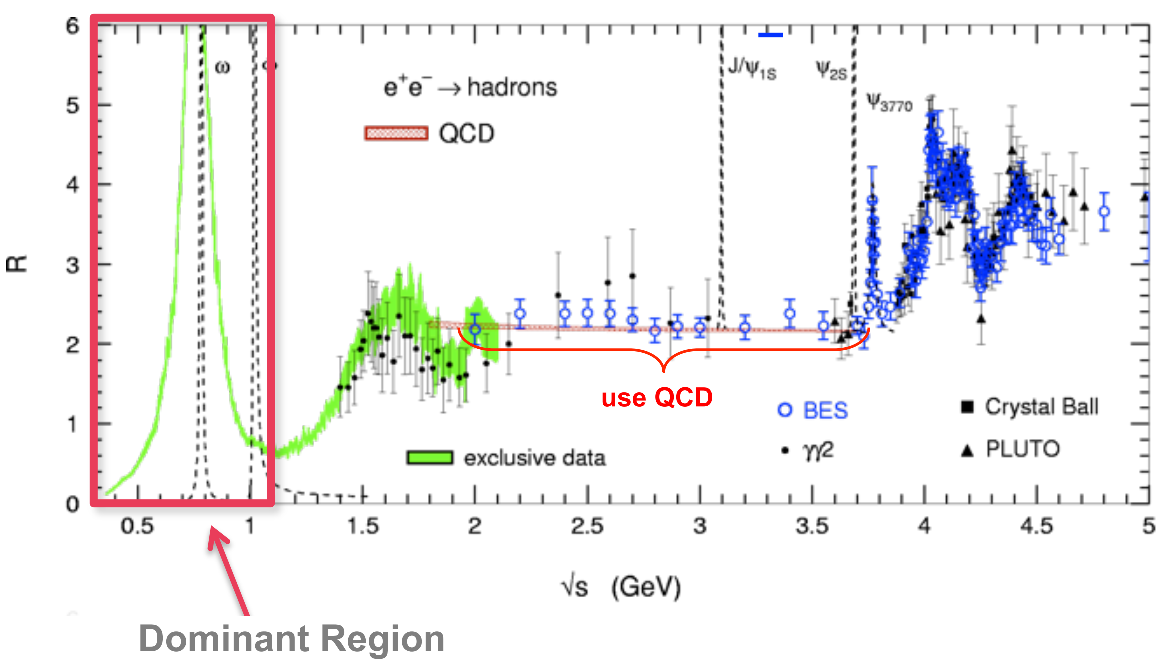
- Leading order hadronic vacuum polarization :

$$a_\mu^{had,LO} = \frac{\alpha^2 m_\mu^2}{(3\pi)^2}\int_{4m_\pi^2}^{\infty} ds\, \frac{K(s)}{s^2} R_V(s)$$

- Low energy contribution dominates : ~75% comes from s < (1 GeV)^2
  ⇒ ππ contribution extracted from data

Computation of HVP using data

- Huge 20-years effort by experimentalists and theorists to reduce error on lowest-order hadronic part
  - ➢ Improved e^+e^- cross section data from Novisibirsk (Russia)
  - ➢ More use of perturbative QCD
  - ➢ Technique of "*radiative return*" allows to use data from Φ and *B* factories
  - ➢ Isospin symmetry allows us to also use τ hadronic spectral functions



But still some progress need to be done

- ➢ Inconsistencies τ vs. e+e-: *Isospin corrections?*
- ➢ Inconsistencies between ISR and direct data: *Radiative corrections?*
- ➢ Lattice Calculation?

New data expected from KLOE2, Belle-II, BES-III?

Computation of HVP using data

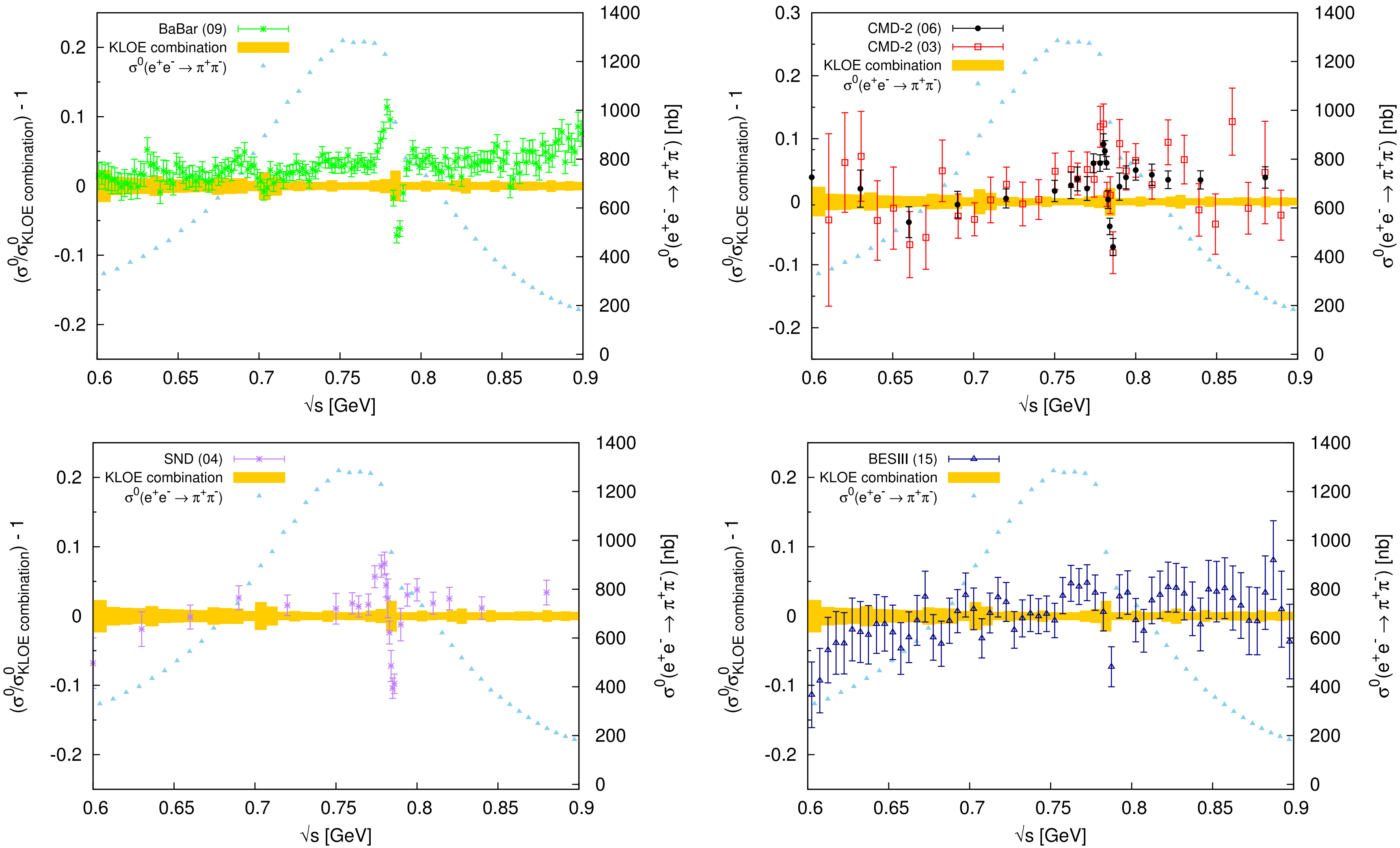
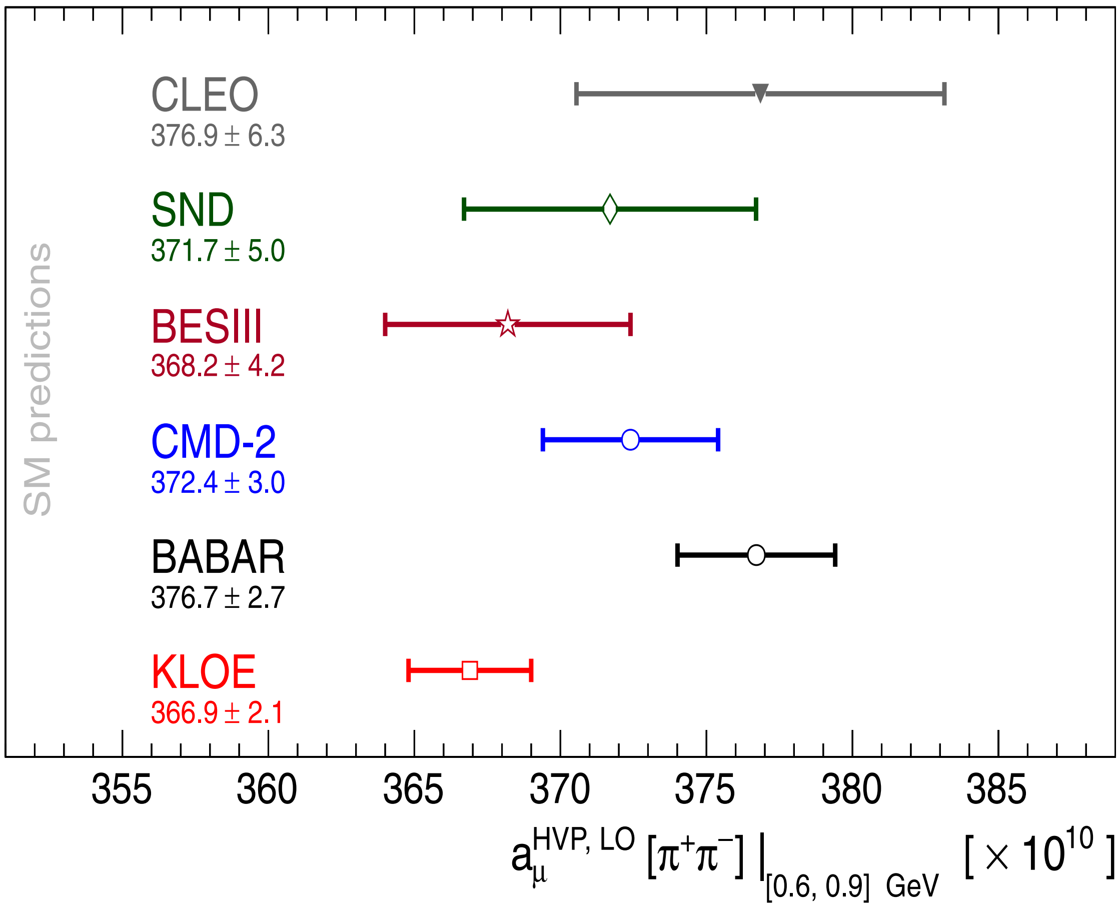


Fig. 13. The $\pi^+\pi$ cross section from the KLOE combination compared to the BABAR, CMD-2, SND, and BESIII data points in the 0.6–0.9 GeV range [82]. The KLOE combination is represented by the yellow band. The uncertainties shown are the diagonal statistical and systematic uncertainties summed in quadrature.

Source: Reprinted from Ref. [82].

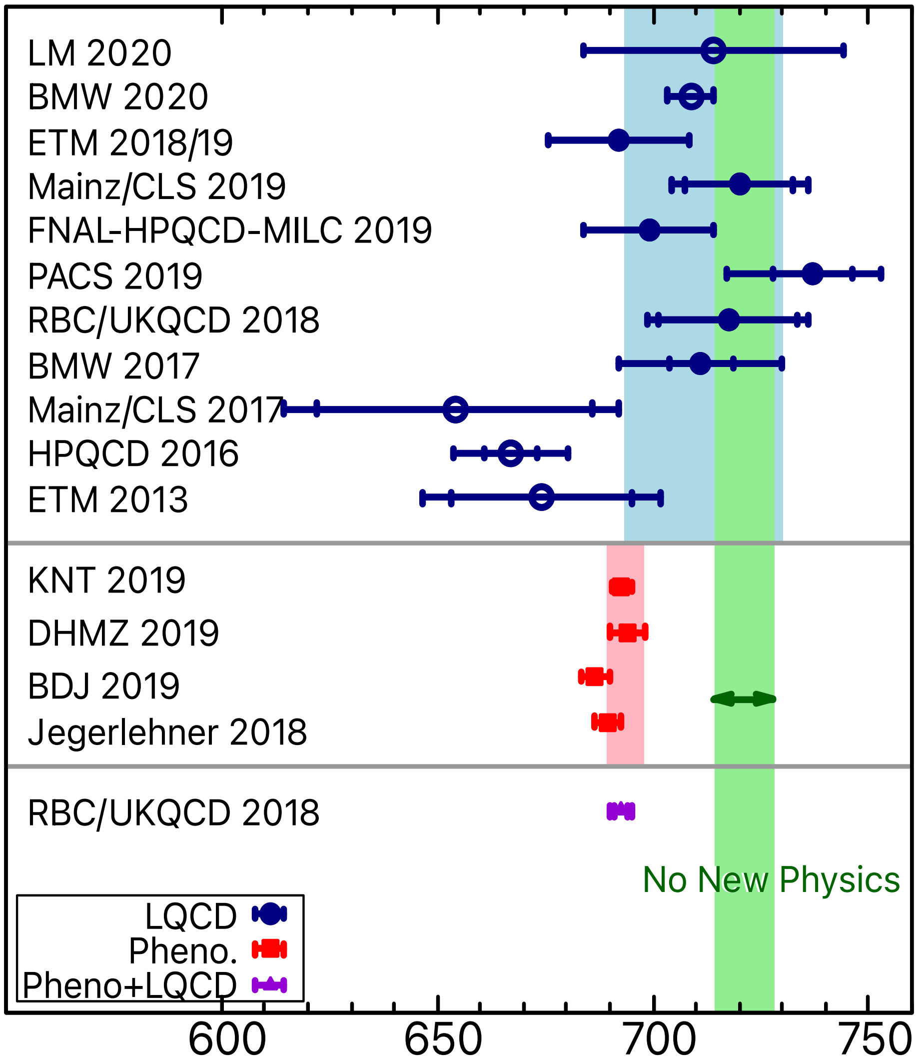
Computation of HVP using data



Comparison of results for $a_\mu^{\text{HVP, LO}}[\pi\pi]$, evaluated between 0.6 GeV and 0.9 GeV for the various experiments..

Computation of HVP using lattice QCD

- Very impressive progress using lattice QCD within the last 5 years



(0.75%)

HVP (BMW-20): $a_\mu = 7087\ (53)$ x 10^{-10}

(2.6%)

HVP (Lattice): $a_\mu = 7116\ (184)$ x 10^{-11}

(0.58%)

HVP (pheno): $a_\mu = 6931\ (40)$ x 10^{-11}

Lattice – pheno $\approx 18.5\ (18.8)$

BMW-20 – pheno $\approx 15.6\ (6.6)$

Computation of HVP using lattice QCD

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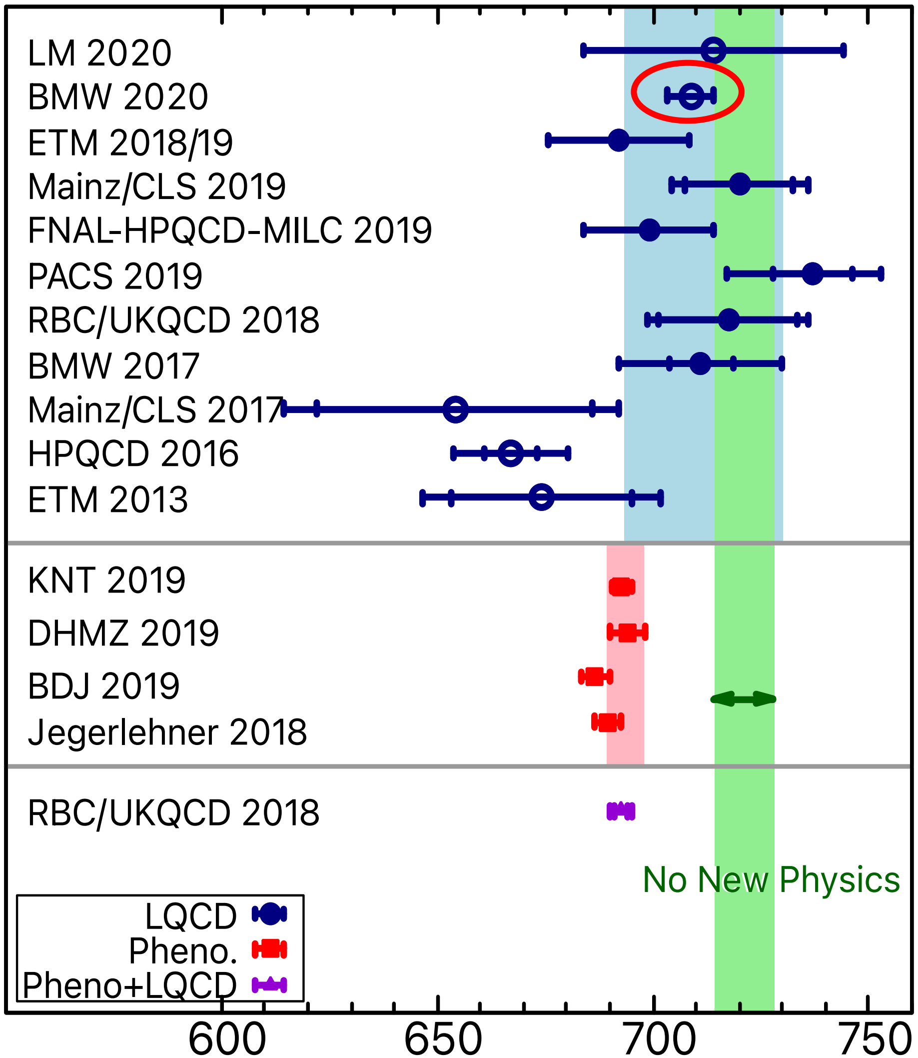

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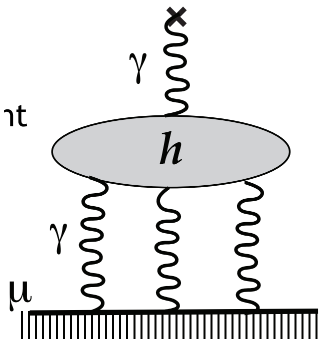
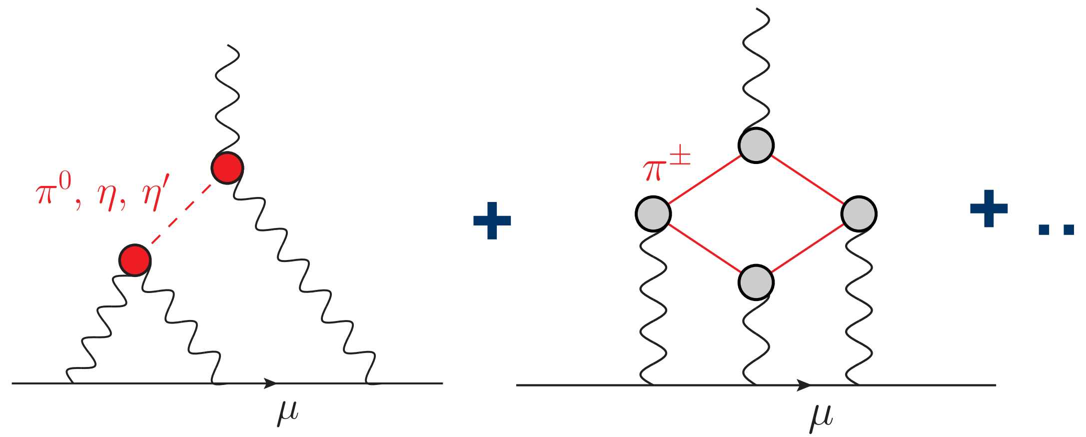
Computation of HLbL using data

g-2 Theory Initiative White Paper'20

- Very impressive progress using dispersive techniques and data in particular in pion pole contribution





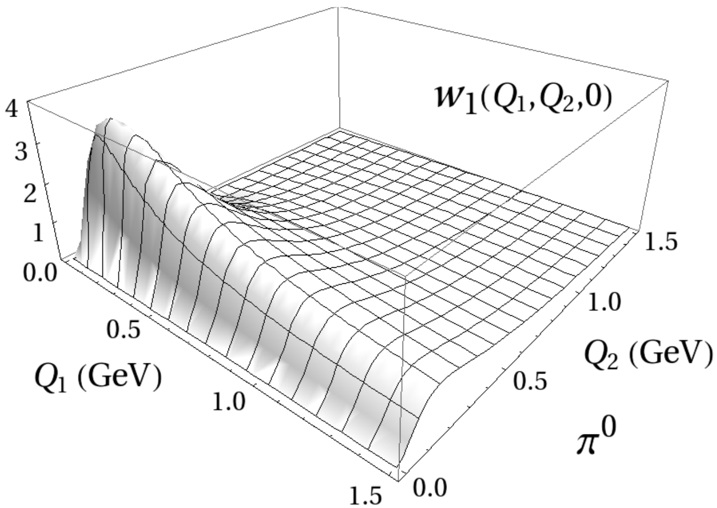
$$a_\mu^{P\text{-pole}} = \left(\frac{\alpha}{\pi}\right)^3 \int dQ_1 dQ_2 d\tau \Big[w_1(Q_1, Q_2, \tau) F_{P\gamma^*\gamma^*}(-Q_1^2, -Q_3^2) F_{P\gamma^*\gamma^*}(-Q_2^2, 0) + w_2(Q_1, Q_2, \tau) F_{P\gamma^*\gamma^*}(-Q_1^2, -Q_2^2) F_{P\gamma^*\gamma^*}(-Q_3^2, 0) \Big]$$

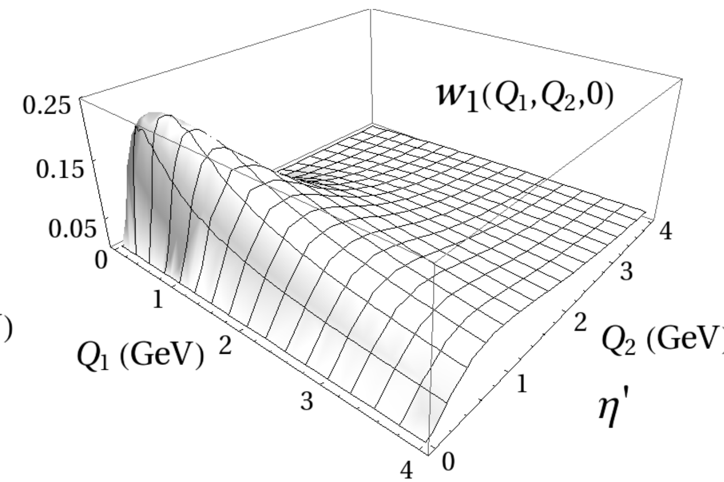
Weight functions

Meson Transitions form factors

Computation of HLbL using data

g-2 Theory Initiative White Paper'20





- Weight contribution ⇒ low energy dominates

$$F_{\pi^0\gamma^*\gamma^*} = F^{\text{disp}}_{\pi^0\gamma^*\gamma^*} + F^{\text{eff}}_{\pi^0\gamma^*\gamma^*} + F^{\text{asym}}_{\pi^0\gamma^*\gamma^*}$$

Use experimental data with dispersive analysis to reconstruct from dominant low-energy singularities (2/3 pions intermediate states)

$$F^{\text{disp}}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = F^{\text{disp}}_{vs}(q_1^2, q_2^2) + F^{\text{disp}}_{vs}(q_2^2, q_1^2)$$

Computation of HLbL using data

g-2 Theory Initiative White Paper'20

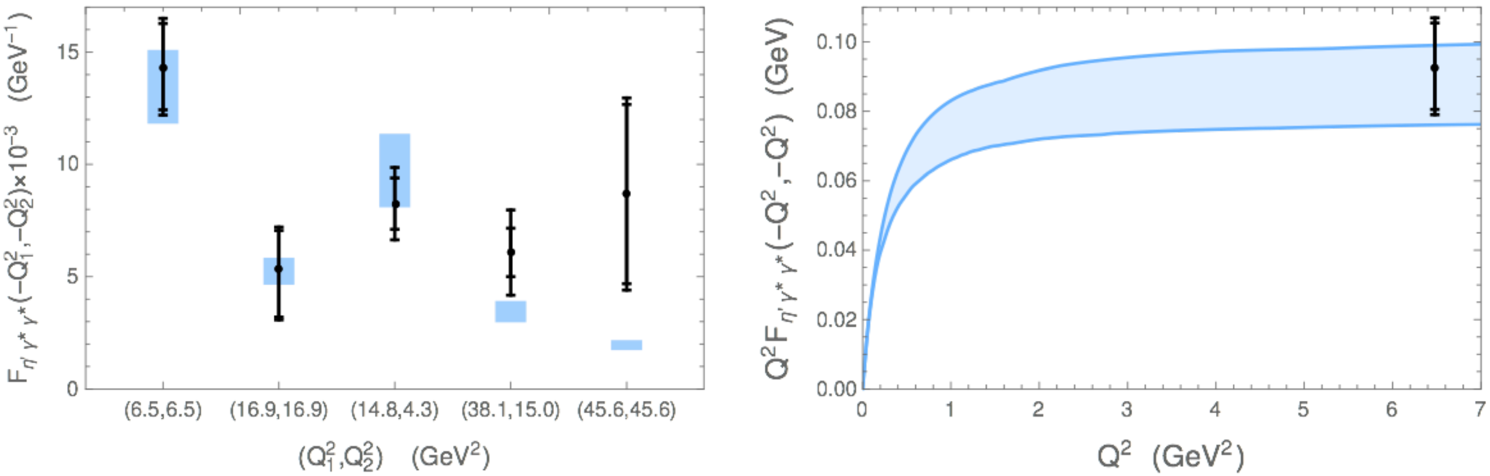
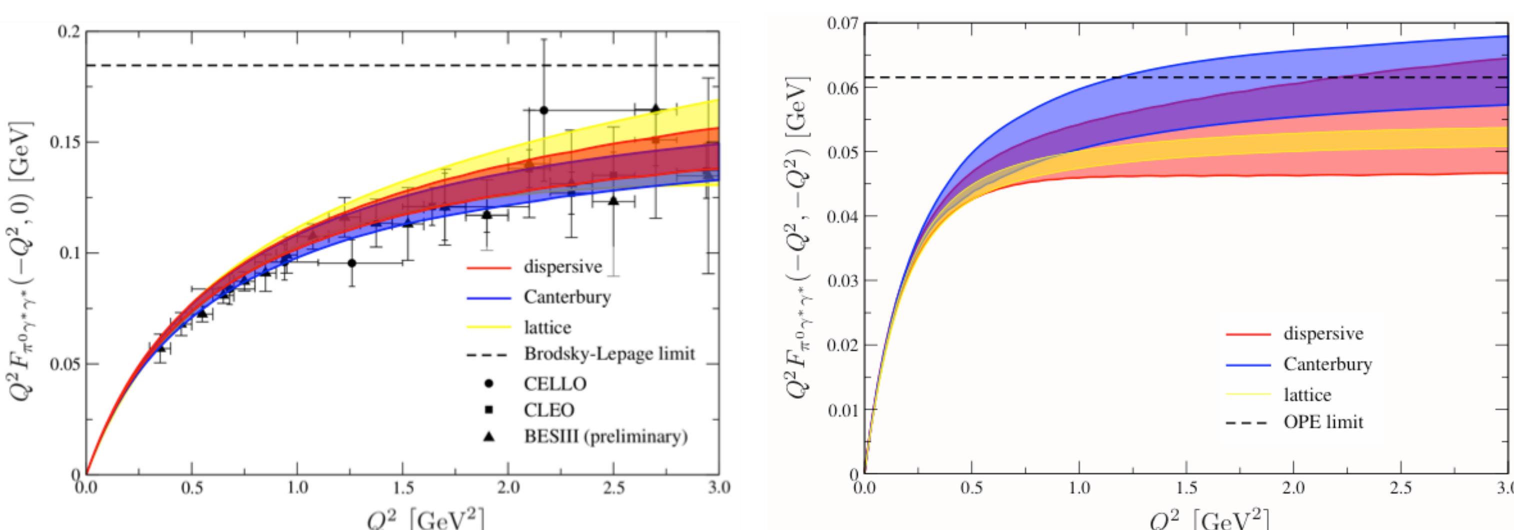


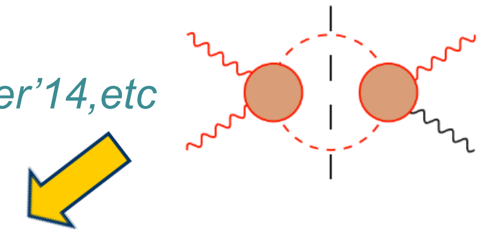
Figure 59: Left: BABAR data points [108] with statistical errors (inner bars) and statistical and systematic combined (outer bars) in black, together with the CA prediction including errors (blue bands). Right: The analogous plot for the diagonal $Q^2F_{\eta'\gamma^*\gamma^*}(-Q^2,-Q^2)$ TFF.

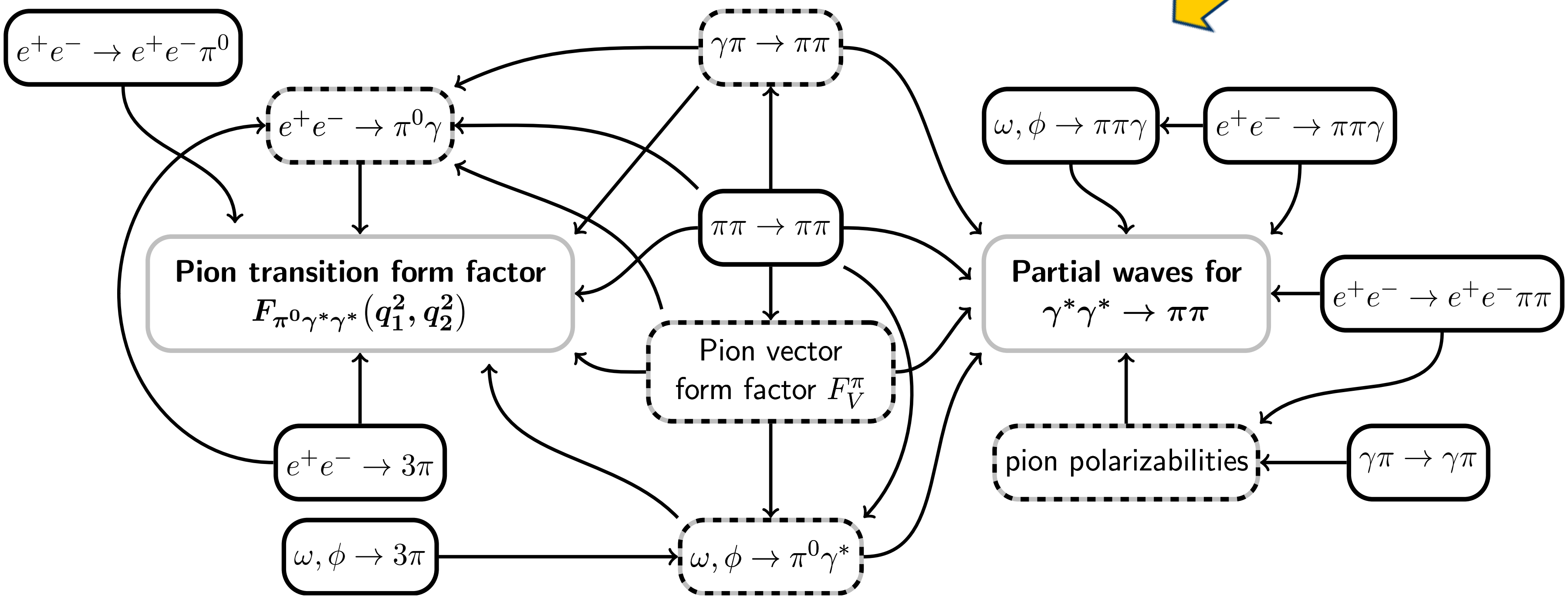


Computation of HLbL using data

- Input for hadronic light-by-light scattering

Colangelo, Hoferichter, Procura, Stoffer'14,etc





Computation of HLbL using data

g-2 Theory Initiative White Paper

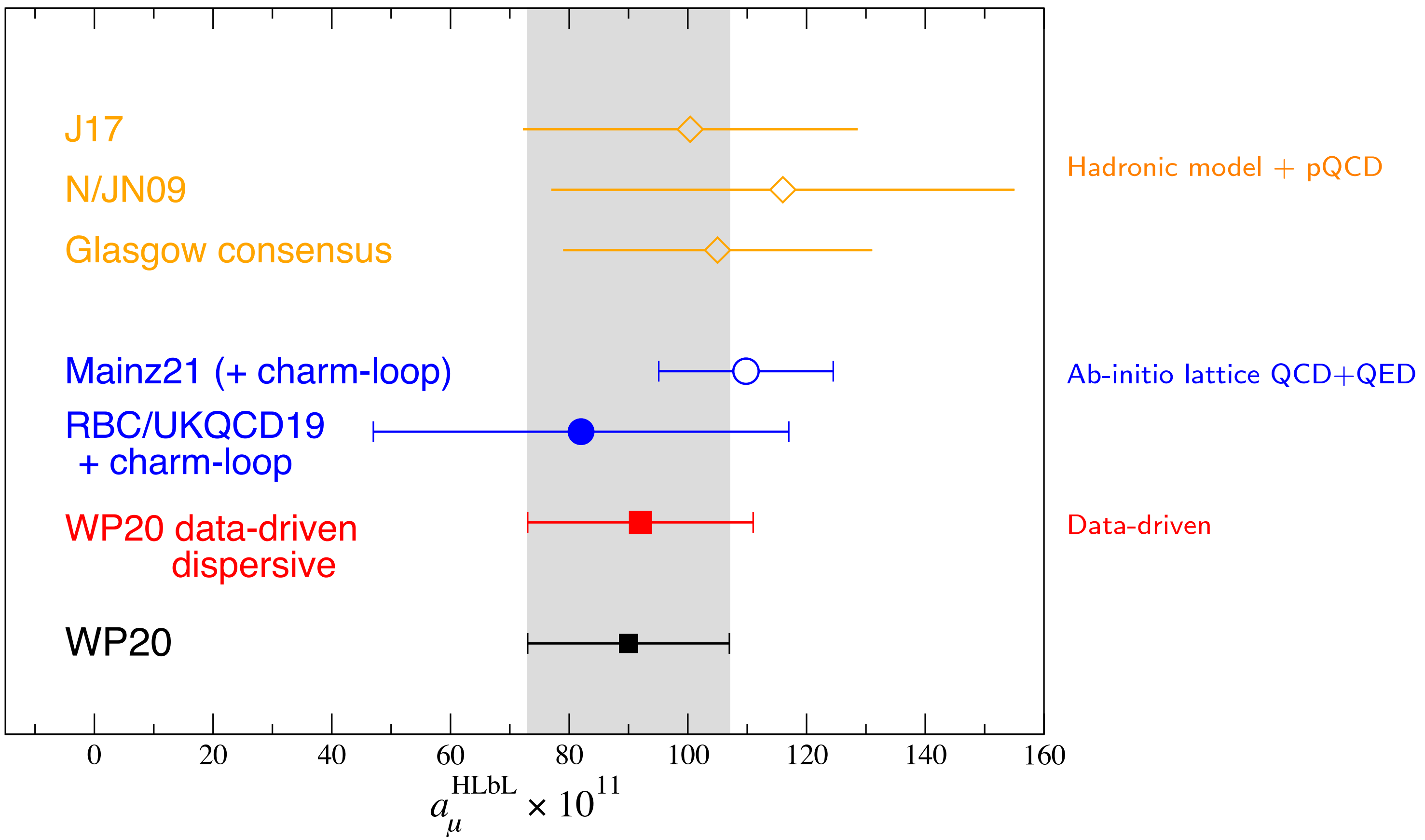
Contribution	PdRV(09) [475]	N/JN(09) [476, 596]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	− 1(3) (scalars + tensors)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

Table 15: Comparison of two frequently used compilations for HLbL in units of 10^{-11} from 2009 and a recent update with our estimate. Legend: PdRV = Prades, de Rafael, Vainshtein ("Glasgow consensus"); N/JN = Nyffeler / Jegerlehner, Nyffeler; J = Jegerlehner.

➡ Very impressive progress since 7 years ago to improve the HLbL determination

Computation of HLbL on the lattice

g-2 Theory Initiative White Paper

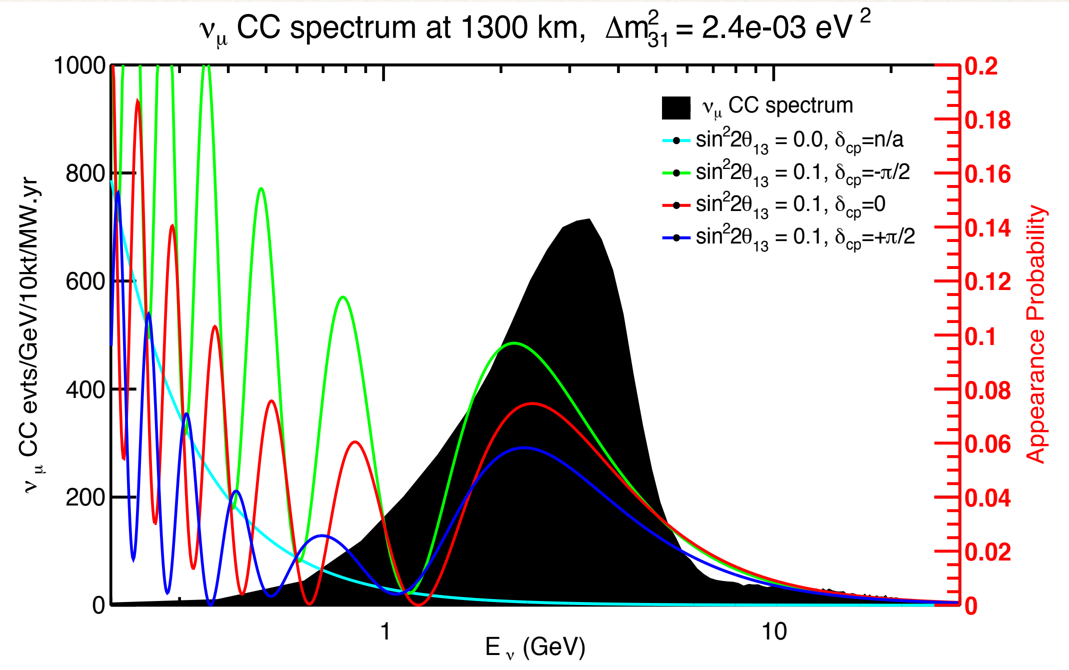


- Not many calculations yet: it is very challenging! The agreement with analytical results is good.

3.3 Axial form factor of the nucleon and neutrino physics

Introduction: Neutrino Cross Section

- Accurate neutrino measurements: *LBNE, Science report'13*
  - Mass hierarchy
  - Oscillations
  - CP violation
  - Beyond 3 flavours?



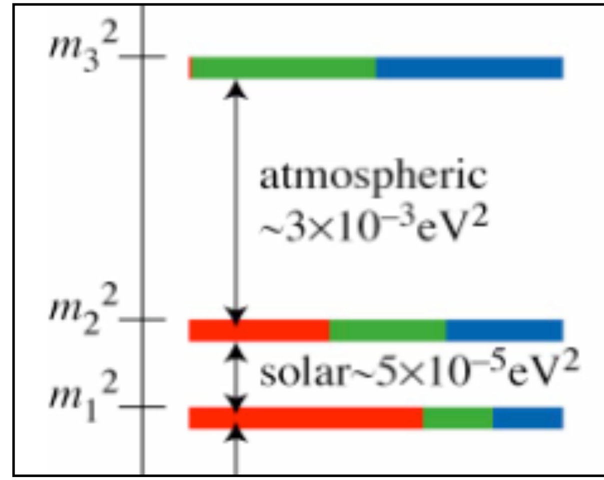
- Precise knowledge of $\boldsymbol{\nu}$ numbers

→ ***Neutrino Cross Section***

→ *New research area developed at IU*

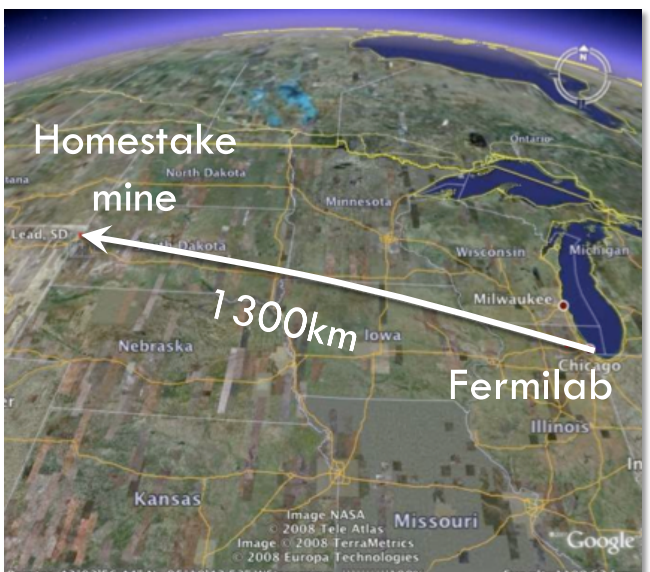
- Need precise E_ν reconstruction

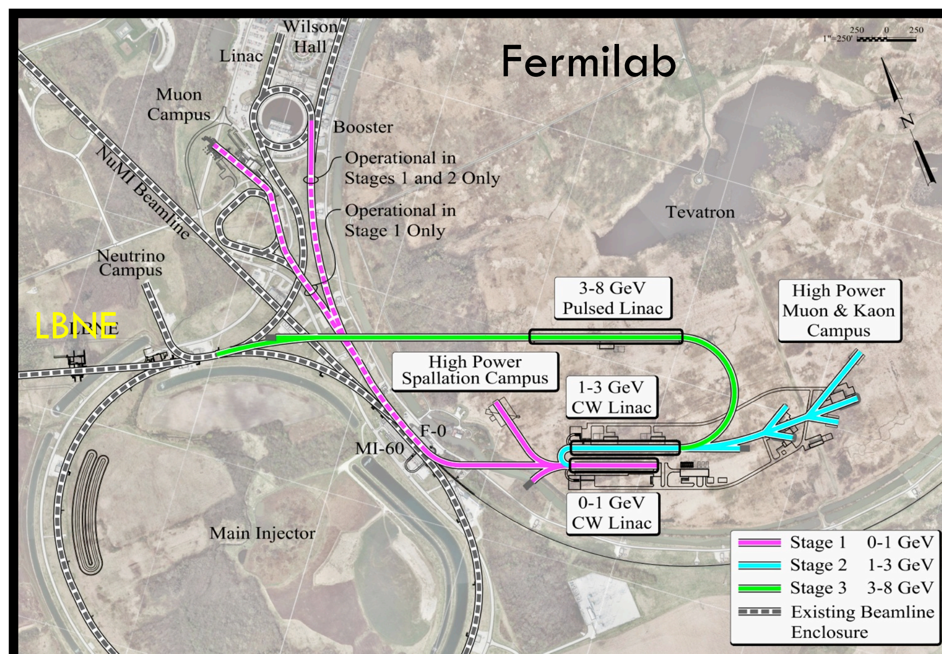
Studying Oscillations



- Experimentally, important effort to study the neutrino oscillations
  ⇨ many experiments in the world!

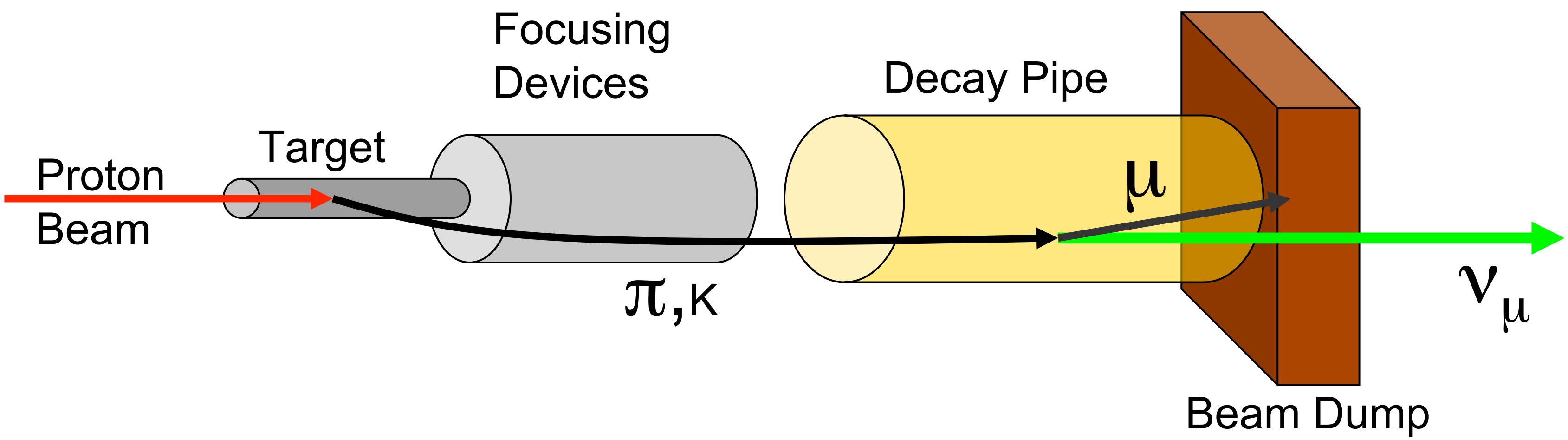
- In the US: a priority ⇨ *LBNE* (*L*ong *B*ase Line *N*eutrino *E*xperiment)
  DUNE (*D*eep *U*nderground *N*eutrino *E*xperiment)



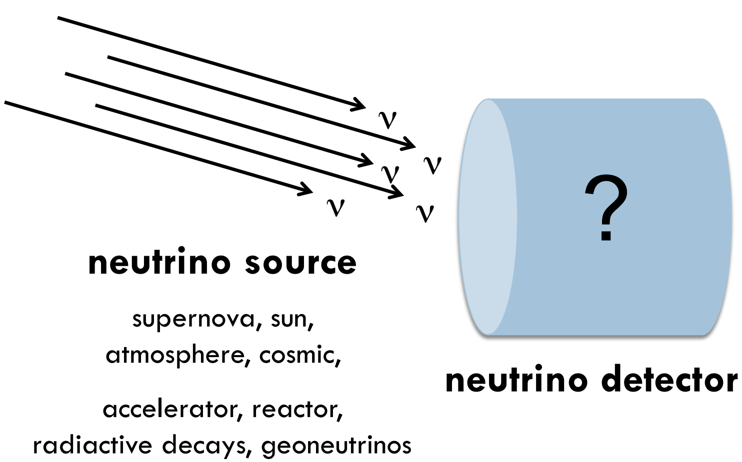


Detection

- Neutrino beams created at Fermilab



- Neutrinos detected 810 miles away in South Dakota in Liquid Argon detectors



$$N_\nu(E) \sim \Phi_\nu(E) \times \sigma_\nu(E) \times \text{target}$$

ν flux

(# neutrinos)
depends on your ν source

at 1 GeV $\sigma(\nu N) \sim 10^{-38}$ cm², compare to $\sigma(pp) \sim 10^{-26}$ cm²

ν cross section

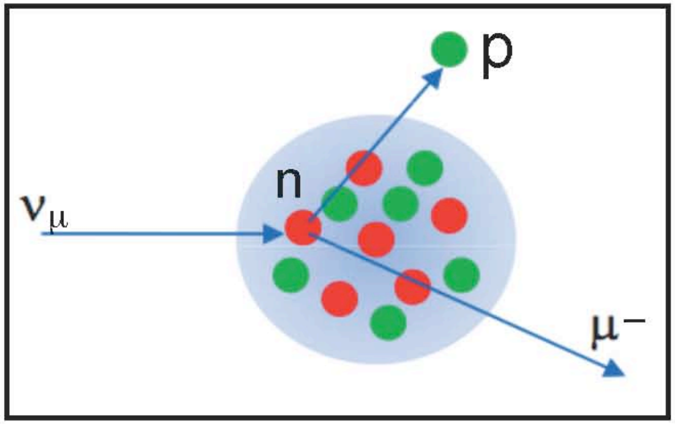
tiny (~10^{-38} cm²)

$\sigma_\nu^{tot} \sim E_\nu$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

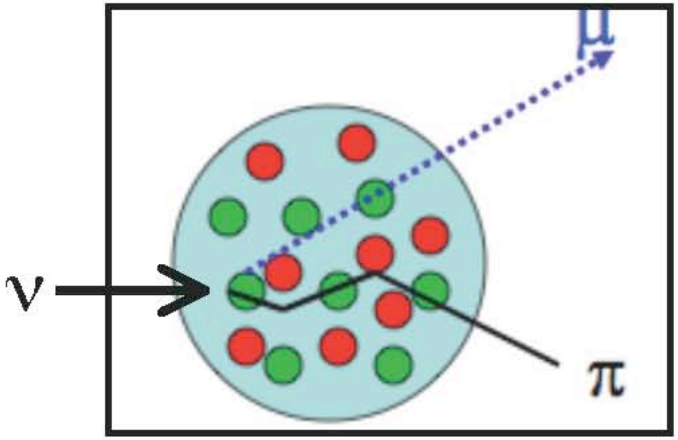
Cross section

- In Liquid Argon detectors interaction of a neutrino with nucleus
  ⇨ Compute cross section



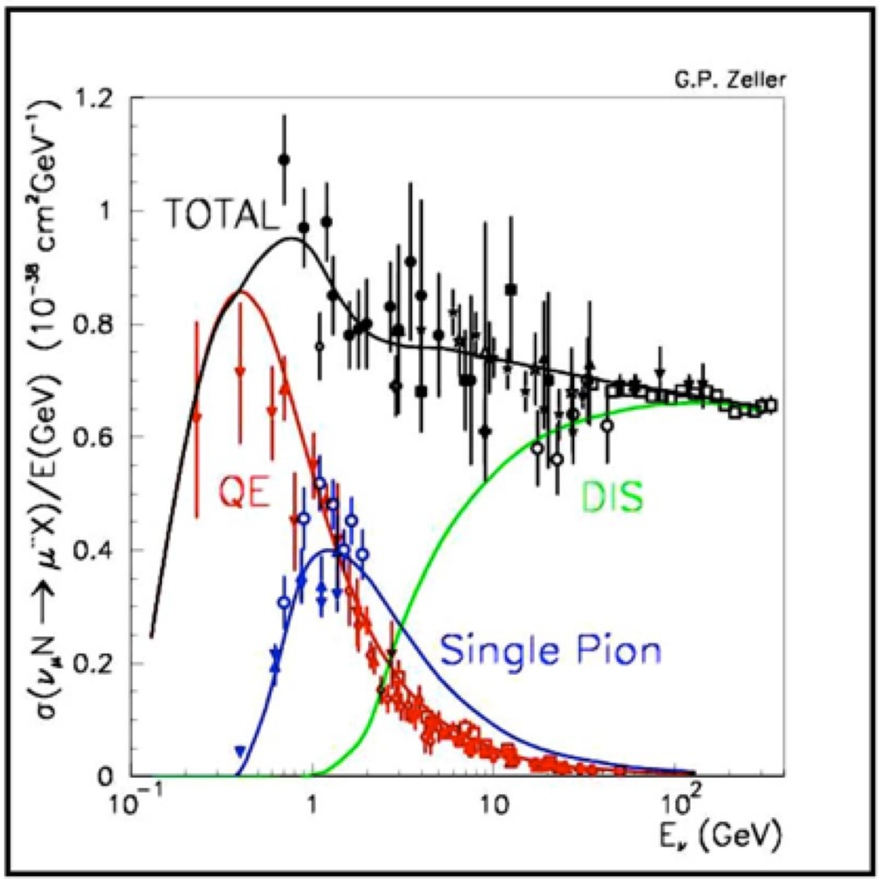
- But not so easy! $E_\nu \sim 1$ GeV
  ⇨ *Non perturbative QCD!*
  Argon, fat nucleus (18 protons, 22 neutrons) several processes:
  - Nucleon form factors
  - Transport inside nucleus

- To be computed for the electron neutrino as well!

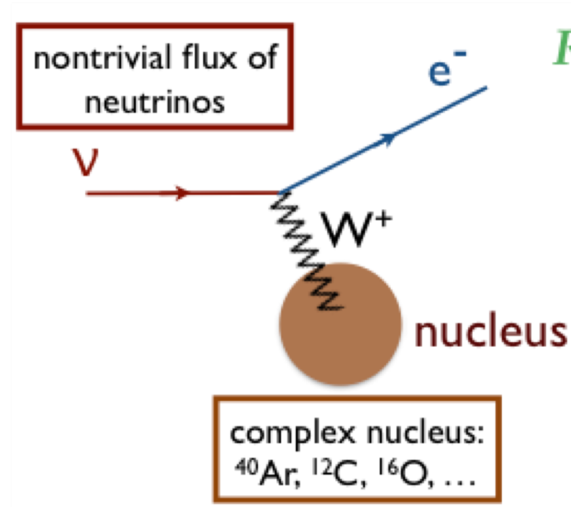


Cross section : different regions

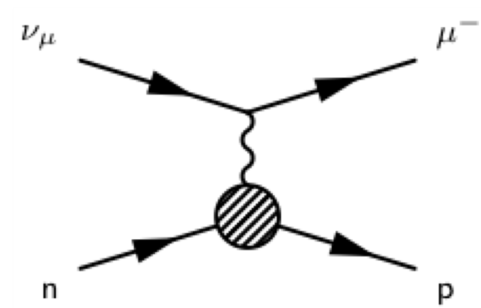
NB: For illustration only



R. Hill

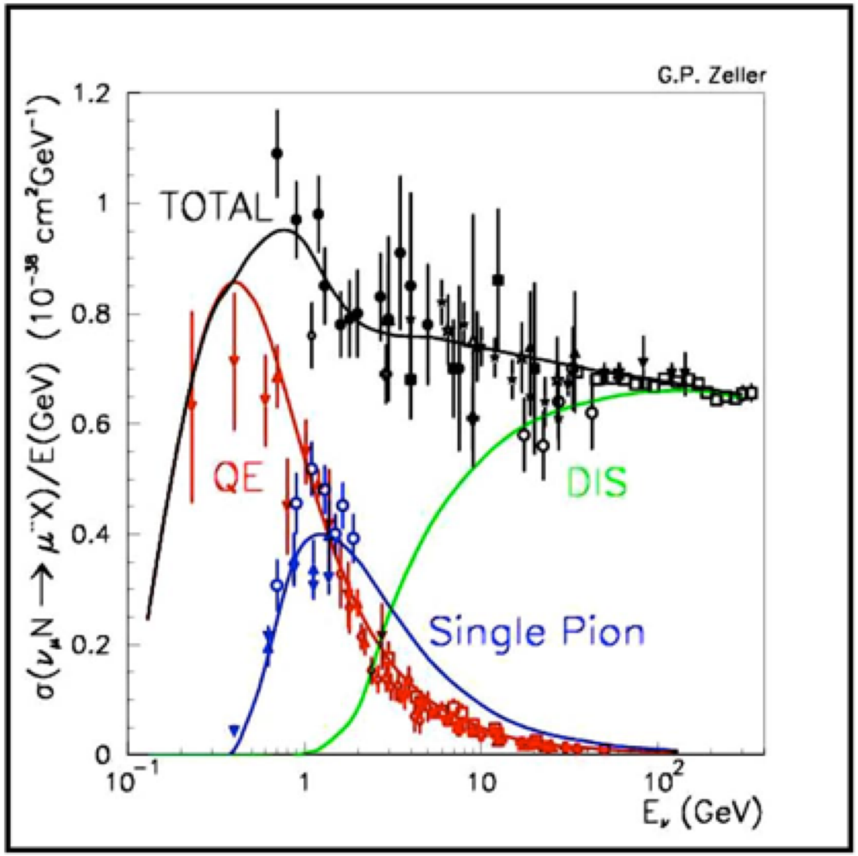


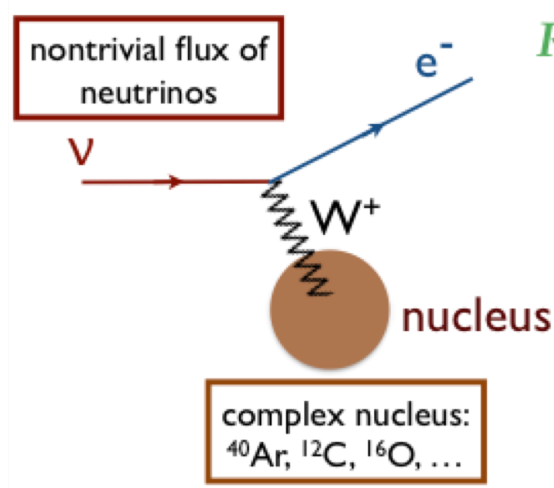
- Different Energy regions:
  - Quasi-Elastic + FSI



Cross section : different regions

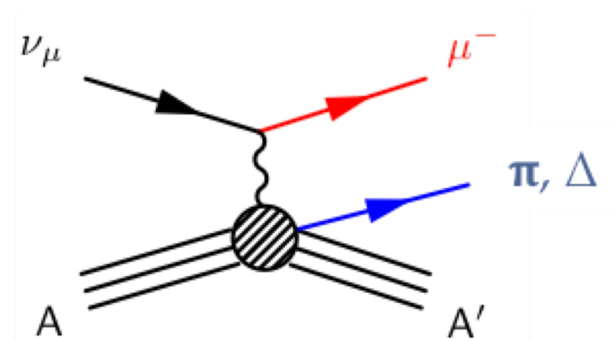
NB: For illustration only





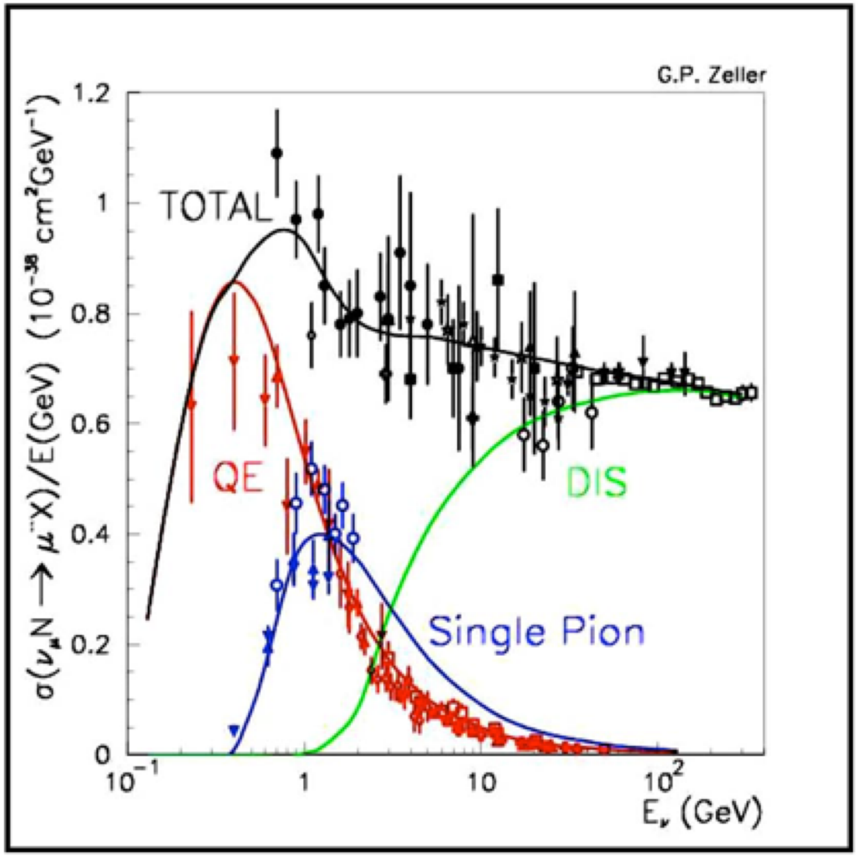
R. Hill

- Different Energy regions:
  - Resonance-pion production

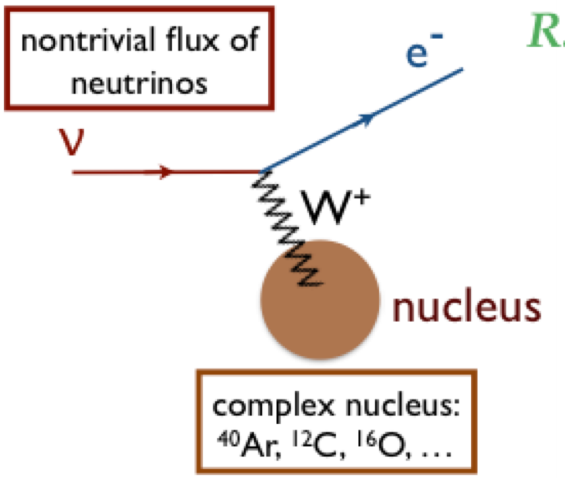


Cross section : different regions

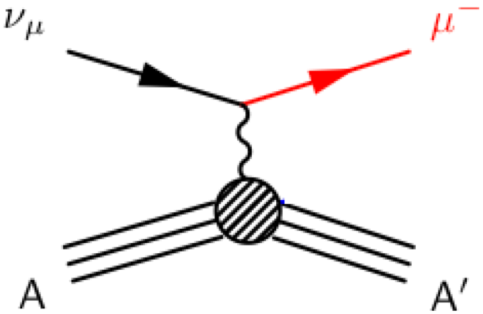
NB: For illustration only



R. Hill



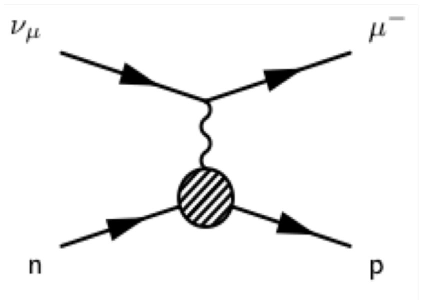
- Different Energy regions:
  - Deep Inelastic scattering

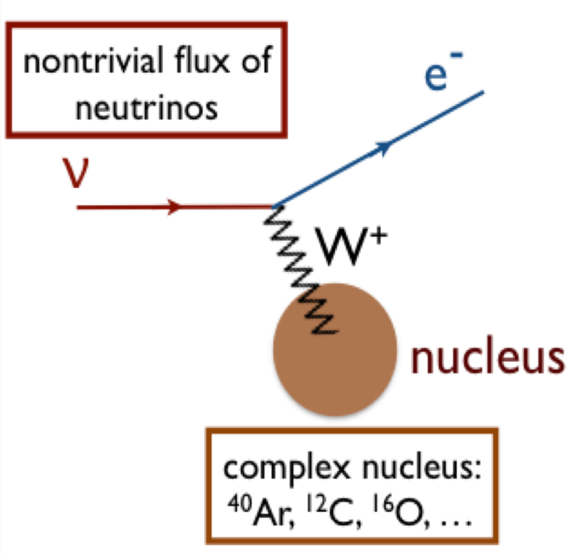


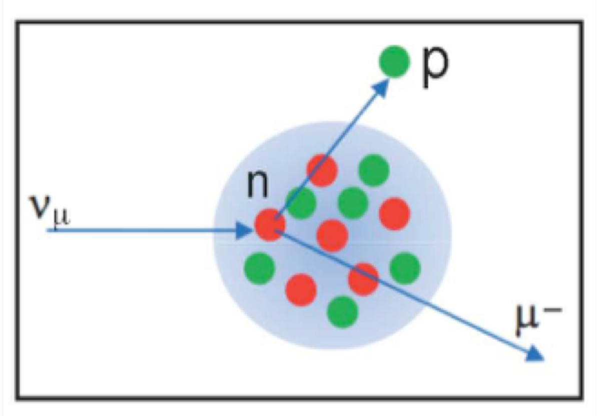
Cross section : different regions

R. Hill

P. Vogel







- Difficulty to describe the hadronization & few body effects and disentangle both effects
  - From quarks to protons and neutrons → *Form factors*
  - From protons and neutrons to nucleus

3.2 Axial form factor and Neutrino Physics

Hadronic matrix element involved:

$$\langle p(p')\,|\,J_W^{+\mu}\,|\,n(p)\rangle \propto \bar{u}^p(p')\left\{\gamma^\mu F_1^V(q^2) + \frac{i}{2m_N}\sigma^{\mu\nu}q_\nu F_2^V(q^2) + \gamma^\mu\gamma_5 F_A(q^2) + \frac{1}{m_N}q^\mu\gamma_5 F_P(q^2)\right\}u^{(n)}(p)$$

- $F_1^V(q^2)$ and $F_2^V(q^2)$ can be extracted from precision electron data at ***Mainz* (*Bernauer et al, A1 coll.'06*)** and ***JLab***
- $F_P(q^2)$ the pseudo-scalar Form Factor is related to $F_A(q^2)$
- The main ***unknown*** is $F_A(q^2)$
  - $F_A(q^2)$ provides the ***largest contribution*** to the QE cross section at 1 GeV

Cannot be determined from electron scattering data

3.2 Axial form factor and Neutrino Physics

- Old problem
- Traditionally it was assumed to follow a simplistic parametrisation

$$F_A(q^2) = \frac{F_A(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2}$$ the dipole parametrisation

- The parameters are $g_A \equiv F_A(0)$ and the axial mass M_A

  → determined using a combination of processes

  - Neutrino nucleon cross section: $\sigma\left(\nu N \to \ell N\right)$
  - Pion electroproduction $\gamma^\star(k^2) + N_1(p_1) \to \pi^a(q) + N_2(p_2)$

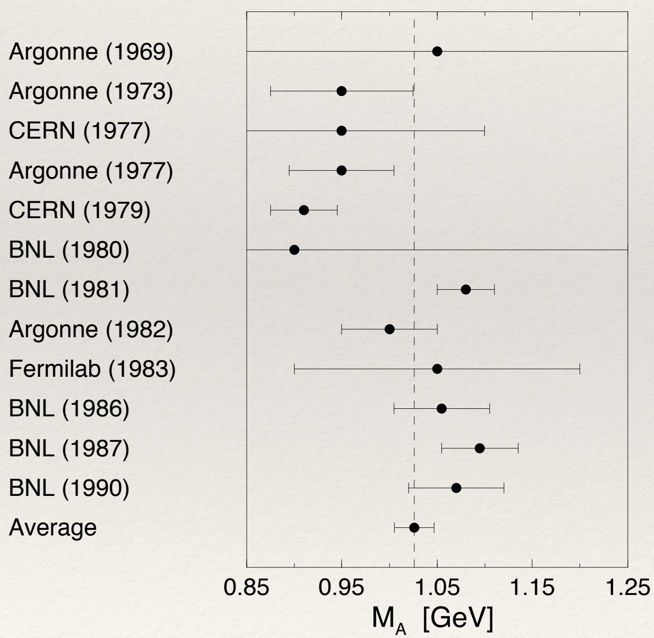
3.2 Axial form factor and Neutrino Physics

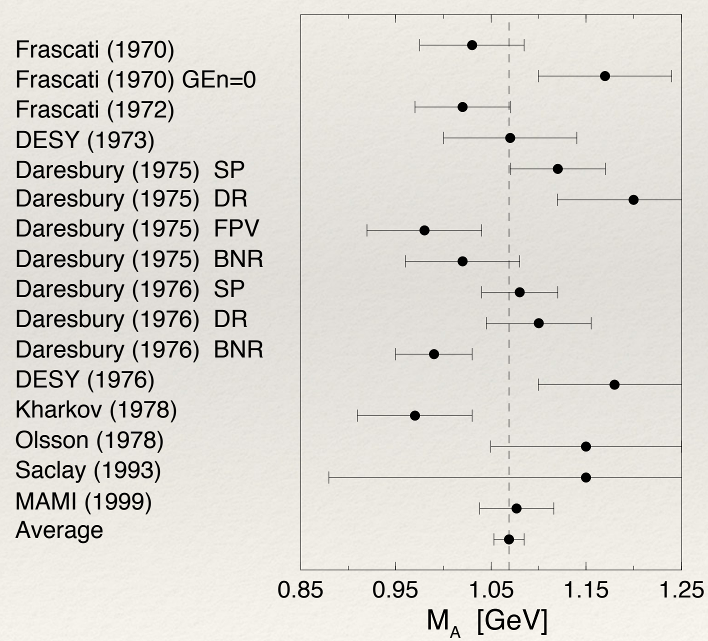
- Up to recently good agreement between all determination of F_A

A. Liesenfeld et al, MAMI'99

$$\nu N \rightarrow \ell N$$

$$\gamma^{\star}(k^2) + N_1(p_1) \rightarrow \pi^a(q) + N_2(p_2)$$



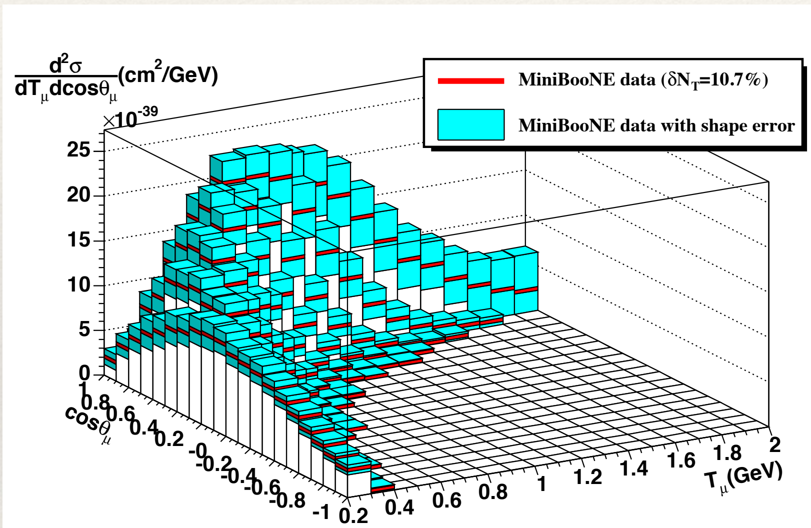


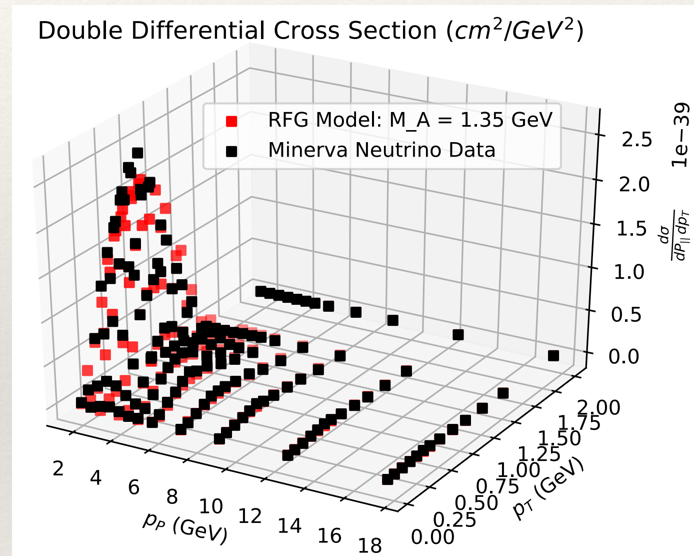
$$M_A = 1.026 \pm 0.021 \text{ GeV}$$

$$M_A = 1.069 \pm 0.016 \text{ GeV}$$

3.2 Axial form factor and Neutrino Physics

- Recently very significant progress on two fronts:
  - Experimentally many new measurements: ***MiniBooNE, K2K, MINERvA, NOMAD***





Reference	m_A [GeV]	$\langle r_A^2 \rangle$ [fm²]
K2K [10]	1.20 ± 0.12	0.32 ± 0.06
NOMAD [11]	1.05 ± 0.06	0.42 ± 0.05
MiniBoonNE [12]	1.35 ± 0.17	0.26 ± 0.06
MINERvA [13]	0.99	0.48
MINOS [14]	$1.23^{+0.13}_{-0.09}$	$0.31^{+0.07}_{-0.05}$

$$F_A(q^2) = F(0)\left(1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \mathcal{O}(q^4)\right)$$

$$\langle r_A^2\rangle = \frac{12}{M_A^2}$$

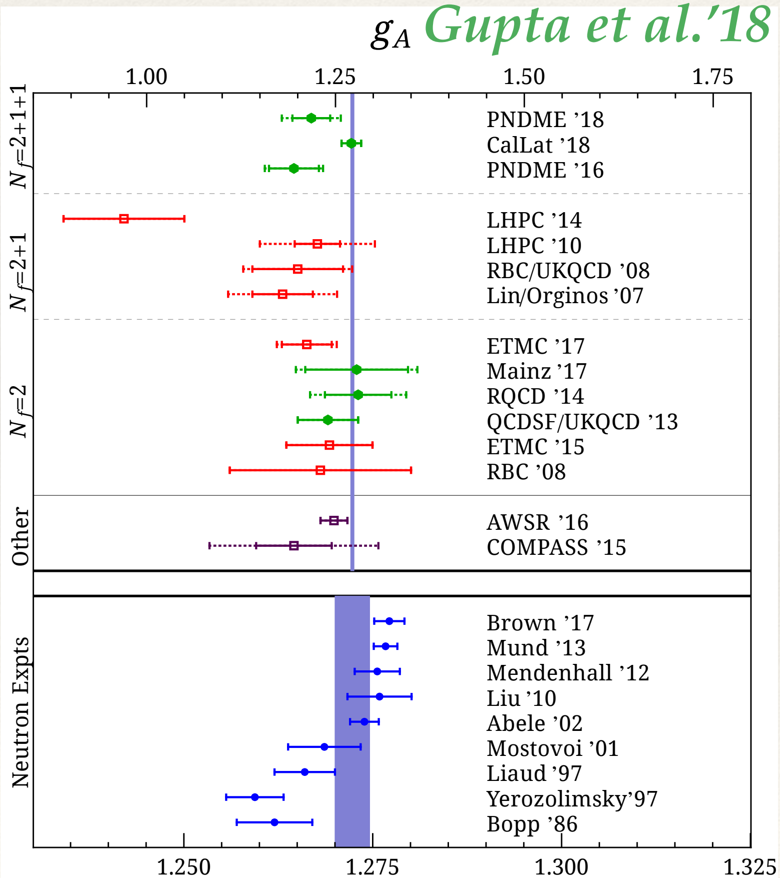
16

3.2 Axial form factor and Neutrino Physics

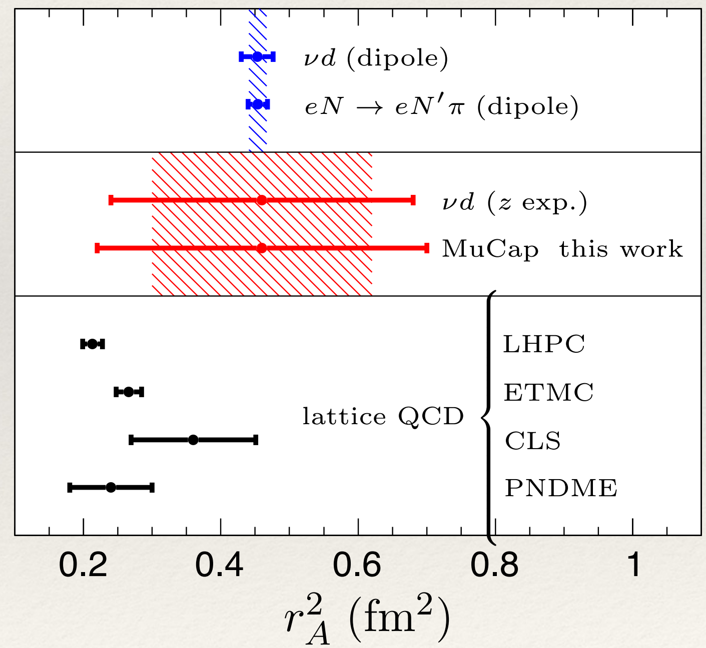
❖ Recently very significant progress on two fronts:

- Lattice QCD results on $g_A \equiv F_A(0)$ and $F_A(q^2)$

Gupta et al.'18

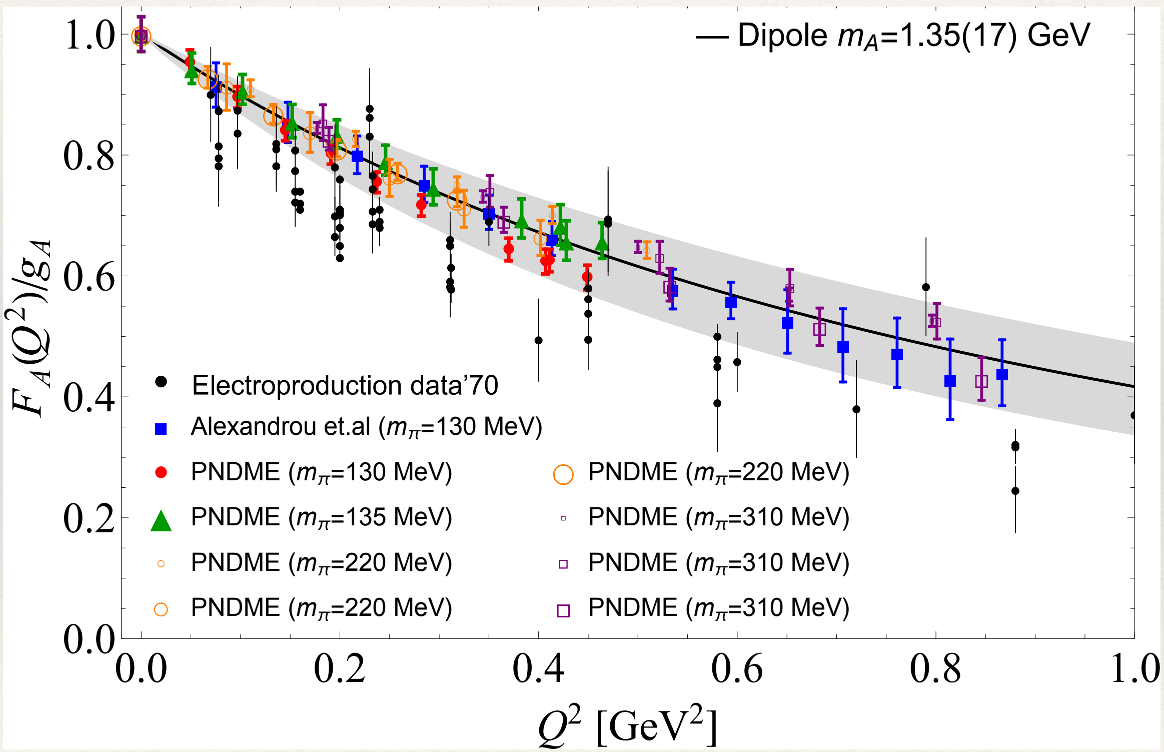


Hill, Kammel, Marciano, and Sirlin'18



3.2 Axial form factor and Neutrino Physics

- Recently very significant progress on two fronts:
  - Lattice QCD results on $g_A \equiv F_A(0)$ and $F_A(Q^2)$



$N_F = 2$ *Alexandrou et al., ETMC'17*

$N_F = 2+1+1$ *Gupta et al., PNDME collab.'17*

see also *Green et al'17, Capitani et al'17*

Bridging Lattice QCD and neutrino measurements

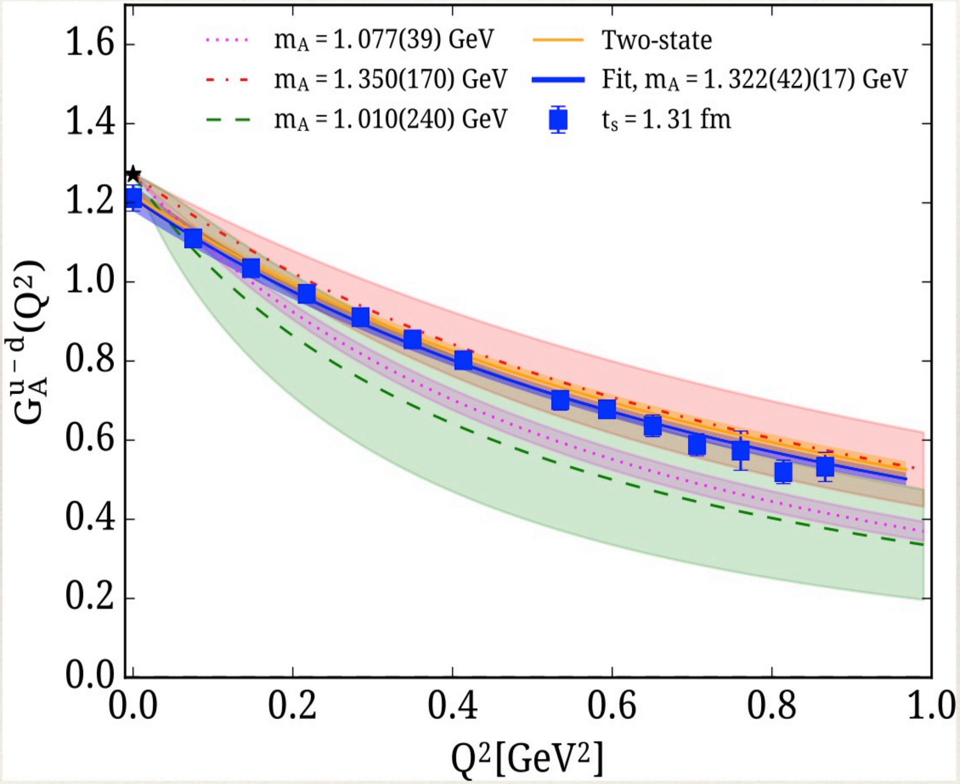
- Connecting predicted $F_A(q^2)$ to measured total and differential cross sections

- Creating a physically motivated analytical parametrisation that can be used to assist and complement the lattice simulations (beyond the dipole)

Friedland, Gonzalez-Solis, E.P., Quirion, Ristow in preparation

Which value of Q^2 impact neutrino data?

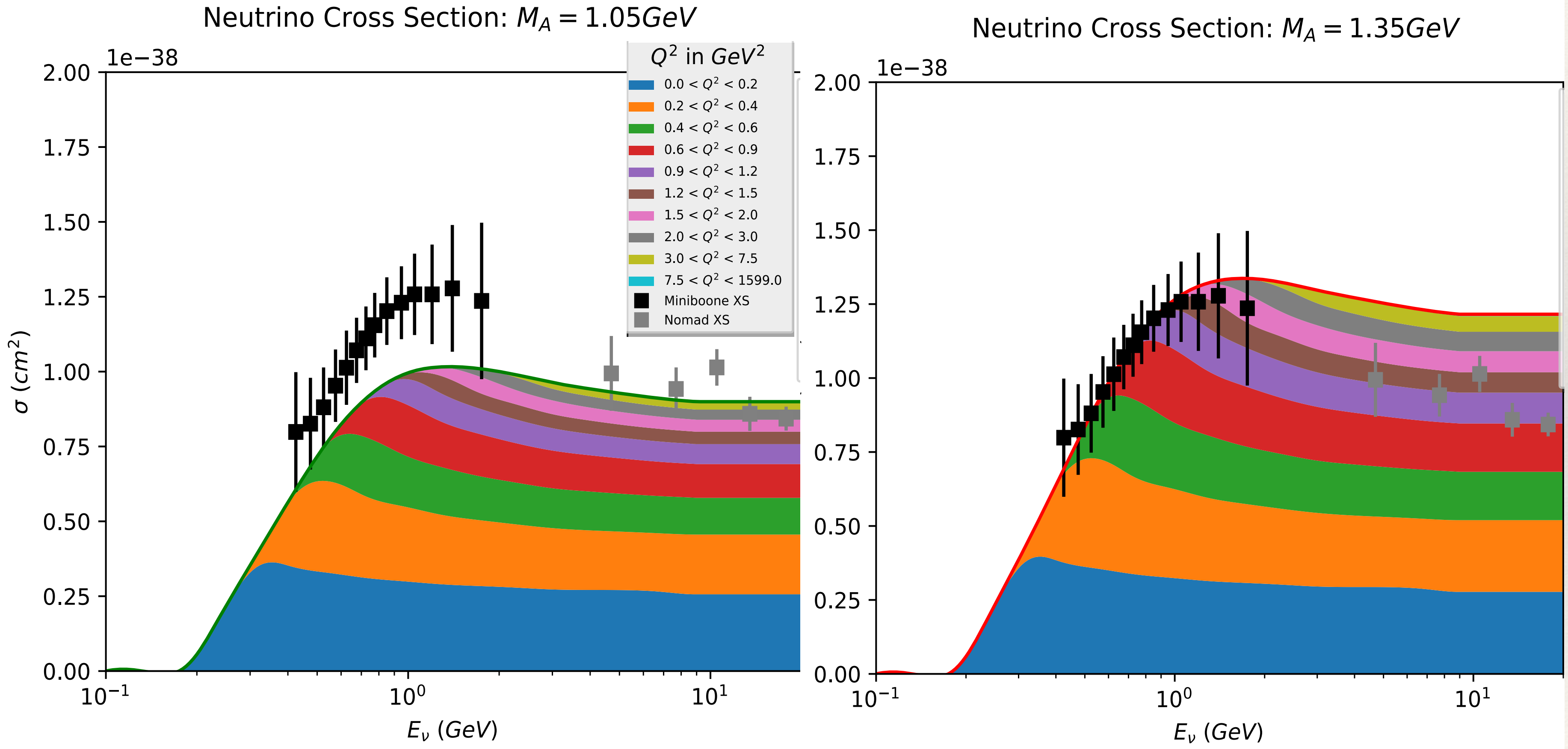
Alexandrou et al., ETMC'17



- Which Q^2 range is important for neutrino XS data?
- If one changes the functional form of F_A, how does that impact the XS prediction?

Which value of Q^2 impact neutrino data?

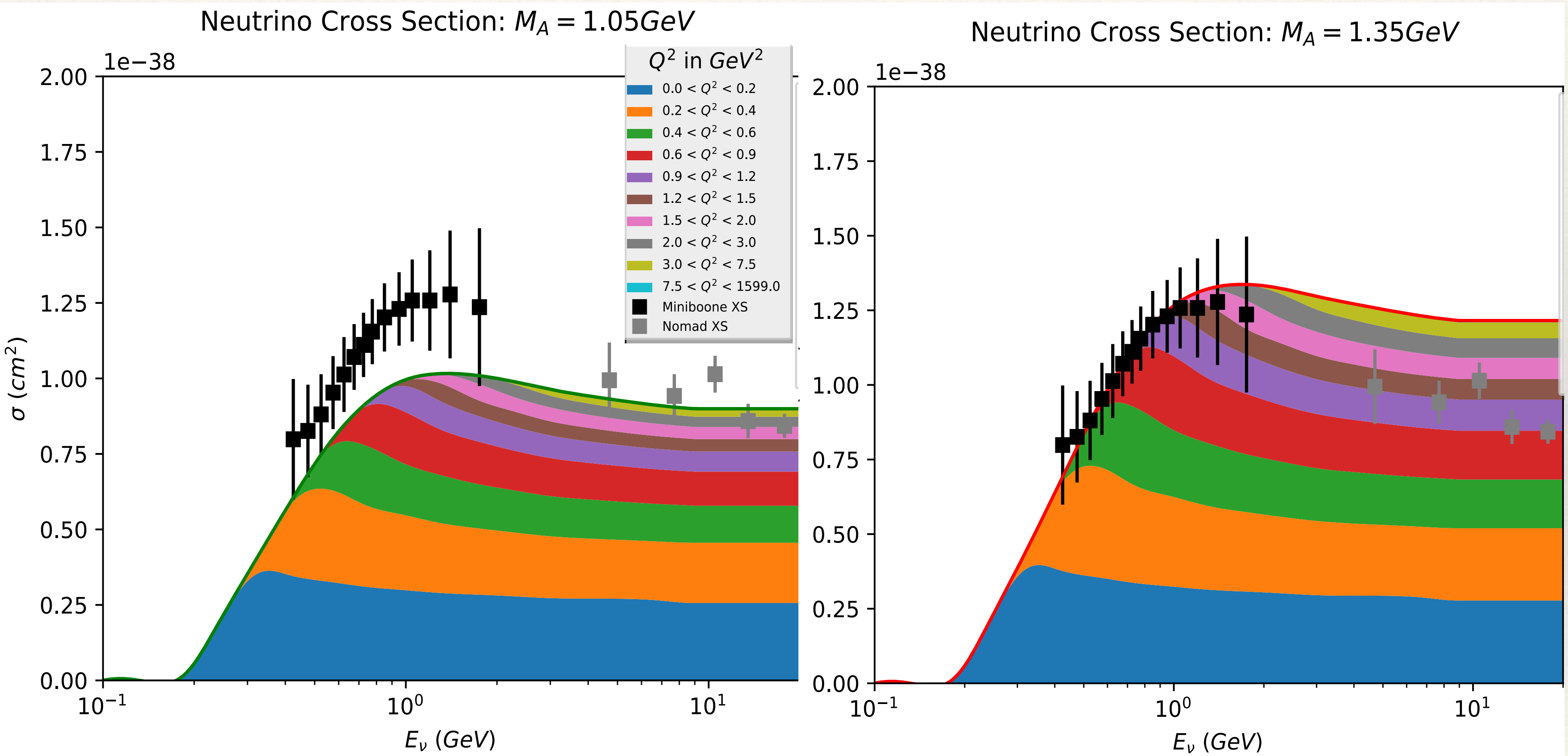
- Composition of MiniBooNE Cross Section



- At E ~ 0.5 GeV the XS comes from $Q^2 < 0.6$ GeV^2

Which value of Q^2 impact neutrino data?

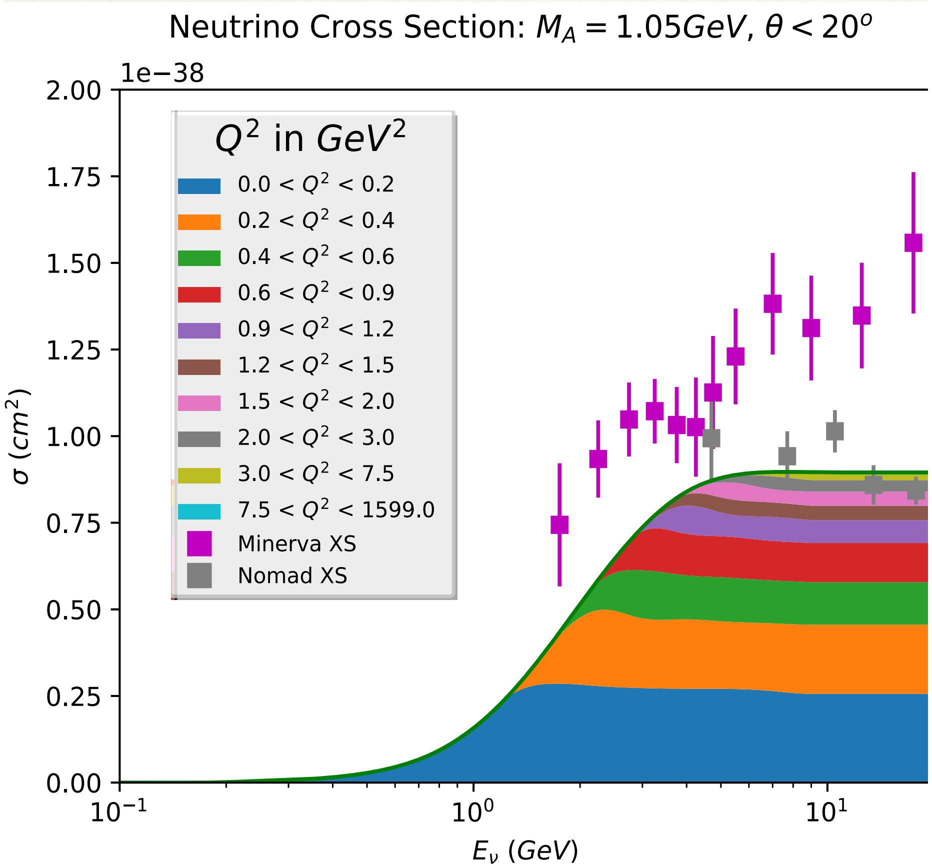
- Composition of MiniBooNE Cross Section

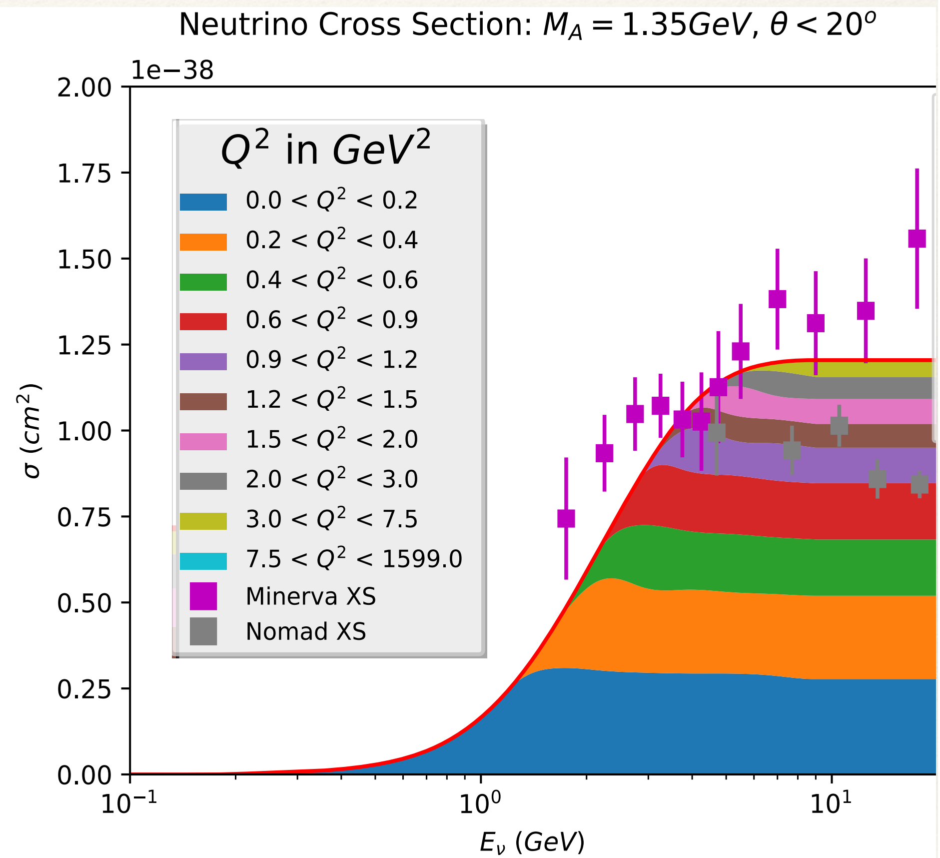


- At E ~ 0.5 GeV the XS comes from $Q^2 < 0.6\ GeV^2$
- At E ~ 1 GeV, ~40% contributions from $0.6\ GeV^2 < Q^2 < 2\ GeV^2$

Which value of Q^2 impact neutrino data?

- Composition of MINERvA Cross Section





- The situation is similar, although E_ν is higher the relevant values are $Q^2 < 2$ GeV^2

Prospects for the future

- Processus involved:

❖ *Quasi elastic scattering*

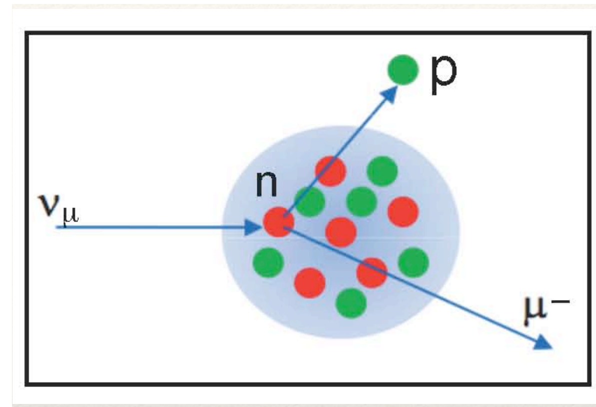
❖ One pion production through resonances

❖ Non-resonant pion production

❖ Deep Inelastic Scattering

❖ Final State Interactions

P. Vogel



- So far we have considered only QE scattering but many more processes involved that need to be understood and requires hadronic physics
  ⇨ multi-year program

4. Conclusion and Outlook

4.1 Conclusion

- Studying fundamental symmetries and testing the Standard Model is crucial to understand fundamental laws of physics and new physics phenomena
- The precision / intensity frontier plays a key role in the search for the “new Standard Model” and its symmetries
- Broad and vibrant experimental program
- K, D and B mesons measurements more accurate → require inputs from hadronic physics
- To reach this quest, studying interactions of quarks, leptons and even neutrinos with high precision requires a precise knowledge of hadronic physics: directly for quark interactions or indirectly for leptons and neutrinos
- We have enter a precision era in all domains of particle physics requiring an unprecedent effort in taming the hadronic uncertainties
- Hadronic physics relies on non-perturbative techniques to treat QCD at low energies: → synergies between lattice QCD and analytical methods: ChPT, dispersion relations, etc.

4.1 Conclusion

- In this lecture, 3 examples:
  - $\eta \to 3\pi$ allows to extract the light quark mass ratios with very good precision
  - Studying the anomalous magnetic moment of the muon
    allows to test the Standard Model very precisely: at the moment there is a discrepancy between SM prediction and experimental measurements. ⇨ We need to work hard on theory front (lattice QCD, analytical methods) and experimental from (g-2 experiment at FNAL and at JPARC) to understand the origin of the discrepancy
    Is it a *hint of New Physics*?
  - To measure the neutrino properties one needs to know the neutrino nucleus cross section with a very good accuracy.

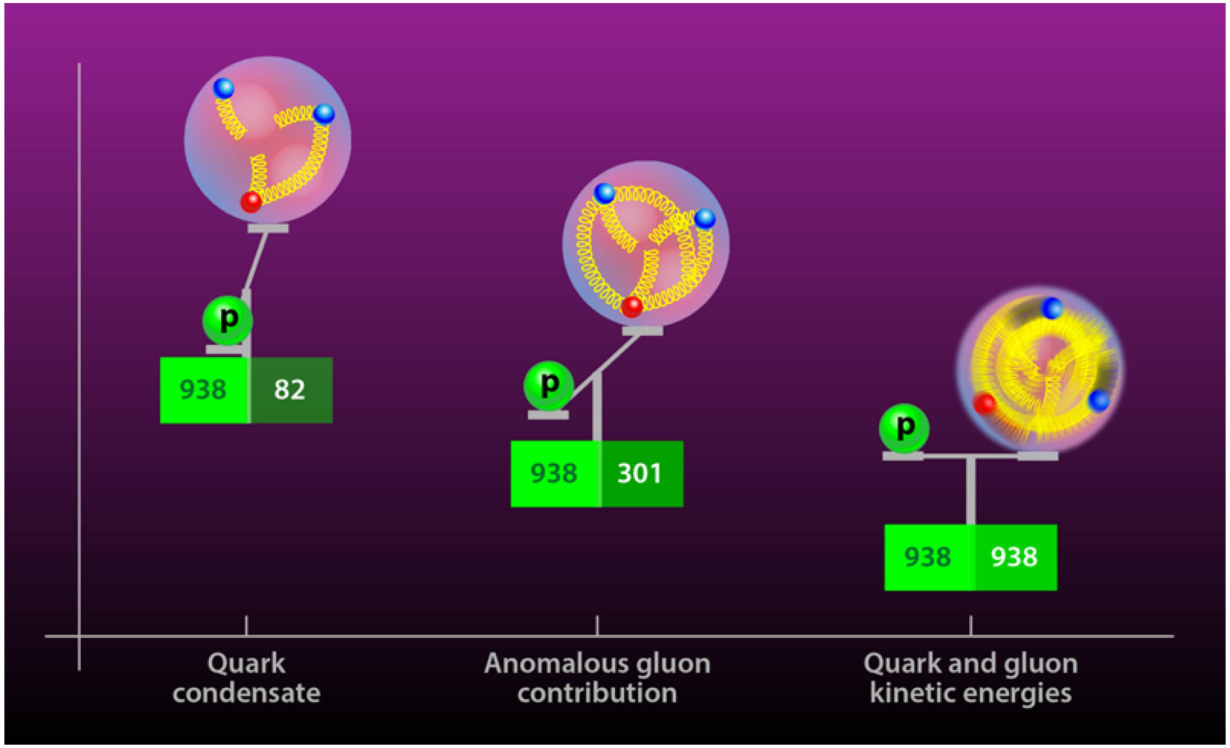
- Many more examples where hadronic physics is of prime importance to be able to interpret the very precise experimental measurements:
  Extraction of CKM mixing parameters, EDMs, Neutrinoless double-beta decays, Neutron decay experiments, …

- The hope is to try to understand the big open questions

3. Back up

Proton

- Let us consider the proton: it is not a fundamental particle, it is made of 3 quarks



Electroweak Interactions: Charged Currents

Experimentally: electroweak interaction exhibits interesting characteristics:

- The doublet partners of the up, charm and top quarks appear to be mixtures of the three quarks with charge − 1/3
  ⇨ the weak eigenstates are different than the mass eigenstates:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak Eigenstates   CKM Matrix   Mass Eigenstates

Unitary 3x3 Matrix, parametrizes rotation between mass and weak interaction eigenstates in Standard Model

Application of EW interactions

- Study of the process: $\nu_e + e^- \rightarrow \nu_e + e^-$
- Can it go through strong, EM, weak interactions?
- How many Feynman diagrams at tree level?

Application of EW interactions

- Study of the process: $\nu_e + e^- \rightarrow \nu_e + e^-$

- Involve leptons only ⇨ no strong interaction

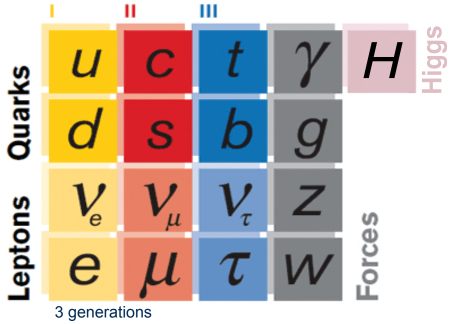
- The neutrinos are electrically neutral ⇨ no EM interaction

  ⇨ Only Weak interactions !

- How many Feynman diagrams?

Description of the weak interactions:

$$\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_\alpha^+ \left(\bar{D}_L V_{CKM} \gamma^\alpha U_L + \bar{e}_L \gamma^\alpha \nu_{e_L} + \bar{\mu}_L \gamma^\alpha \nu_{\mu_L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau_L} \right) + \text{h.c.}$$



Probing the CKM mechanism

- The CKM Mechanism source of *Charge Parity Violation* in SM
- Unitary 3x3 Matrix, parametrizes rotation between mass and weak interaction eigenstates in Standard Model

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak Eigenstates CKM Matrix Mass Eigenstates

$$\sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

3.1 Probing the CKM mechanism

- The CKM Mechanism source of *Charge Parity Violation* in SM
- Unitary 3x3 Matrix, parametrizes rotation between mass and weak interaction eigenstates in Standard Model

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak Eigenstates CKM Matrix Mass Eigenstates

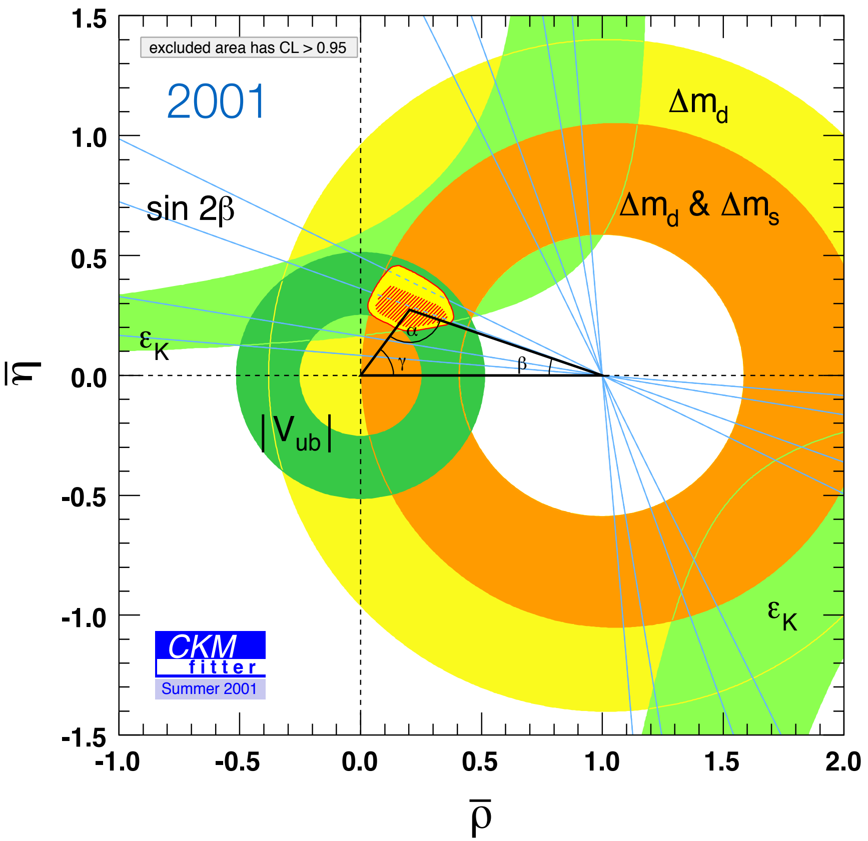
- Fully parametrized by **four** parameters if unitarity holds: three real parameters and *one complex phase* that if non-zero results in *CPV*
- Unitarity can be visualized using triangle equations, e.g.

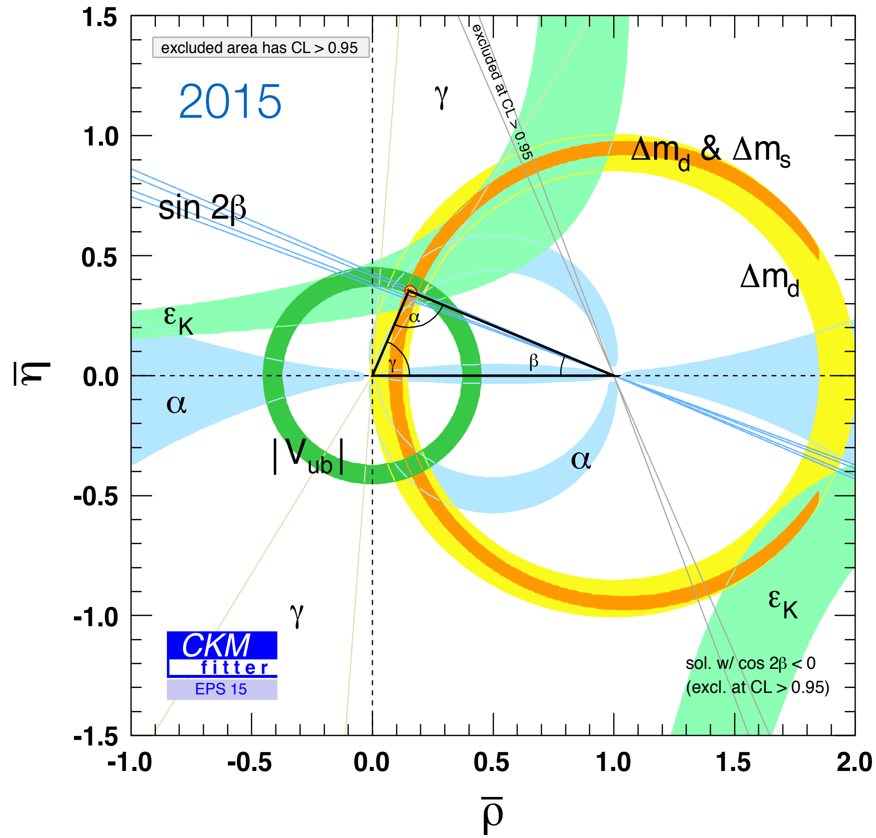
$$V_{CKM}V_{CKM}^{\dagger} = \mathbf{1} \quad \rightarrow \quad \boxed{V_{ub}^{*}V_{ud} + V_{cb}^{*}V_{cd} + V_{tb}^{*}V_{td} = 0}$$

CKM picture over the years: from discovery to precision

Existence of *CPV* phase established in 2001 by BaBar & Belle

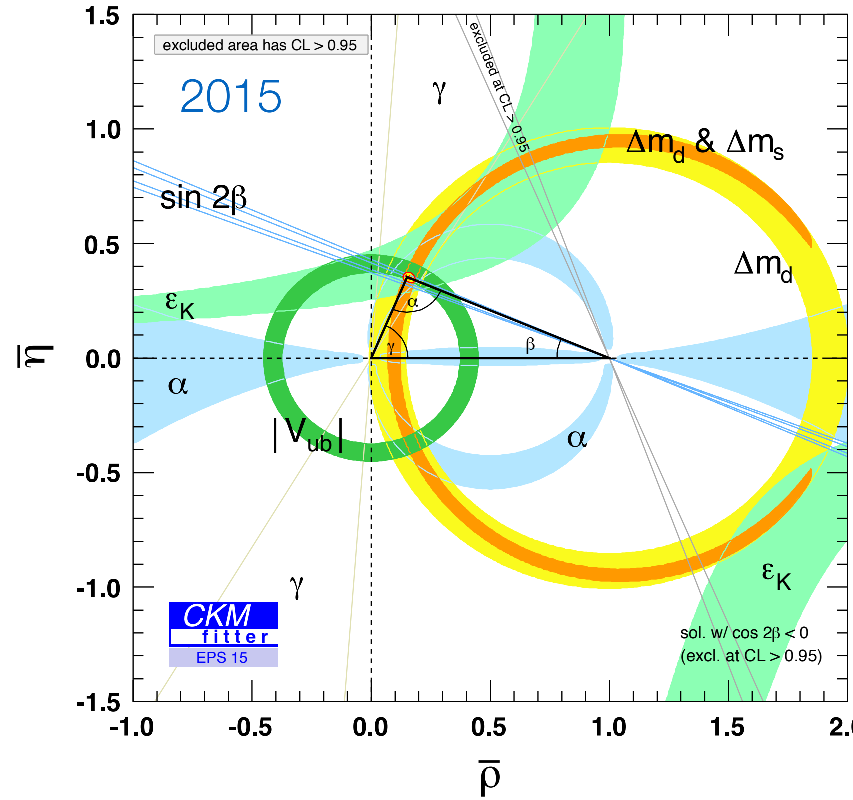
- Picture still holds 15 years later, constrained with remarkable precision
- But: still leaves room for new physics contributions





3.1 Probing the CKM mechanism

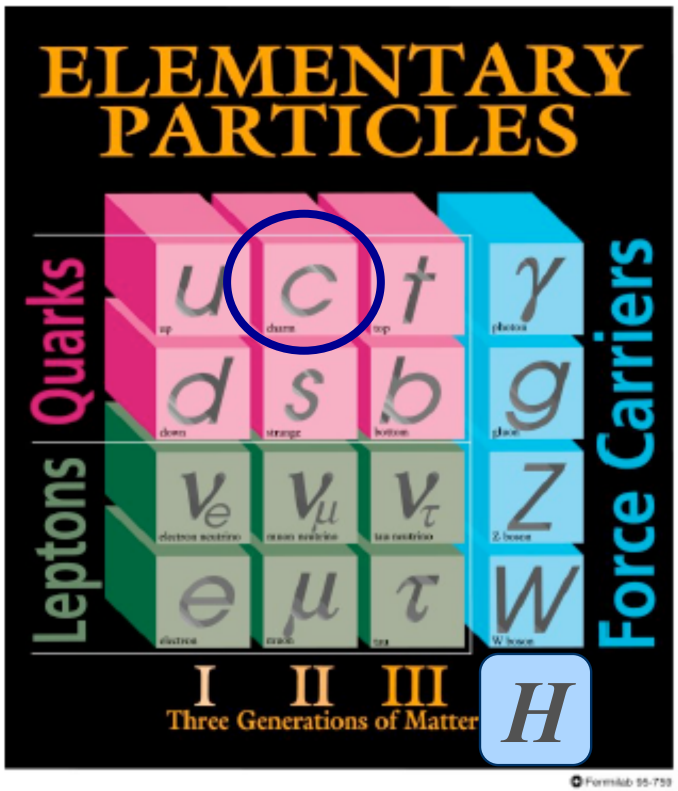
Input	World average 2016	Belle II (+LHCb) 2025
$\lvert V_{ub}\rvert$(semileptonic)[10^{-3}]	$4.01 \pm 0.08 \pm 0.22$	± 0.10
$\lvert V_{cb}\rvert$(semileptonic)[10^{-3}]	$41.00 \pm 0.33 \pm 0.74$	± 0.57
$\mathcal{B}(B \to \tau\nu)$	1.08 ± 0.21	± 0.04
$\sin 2\beta$	0.691 ± 0.017	± 0.008
$\gamma[°]$	$73.2^{+6.3}_{-7.0}$	± 1.5 (± 1.0)
$\alpha[°]$	$87.6^{+3.5}_{-3.3}$	± 1.0
Δm_d	0.510 ± 0.003	-
Δm_s	17.757 ± 0.021	-
$\mathcal{B}(B_s \to \mu\mu)$	$2.8^{+0.7}_{-0.6}$	(± 0.5)
f_{B_s}	$0.224 \pm 0.001 \pm 0.002$	0.001
B_{B_s}	$1.320 \pm 0.016 \pm 0.030$	0.010
f_{B_s}/f_{B_d}	$1.205 \pm 0.003 \pm 0.006$	0.005
B_{B_s}/B_{B_d}	$1.023 \pm 0.013 \pm 0.014$	0.005

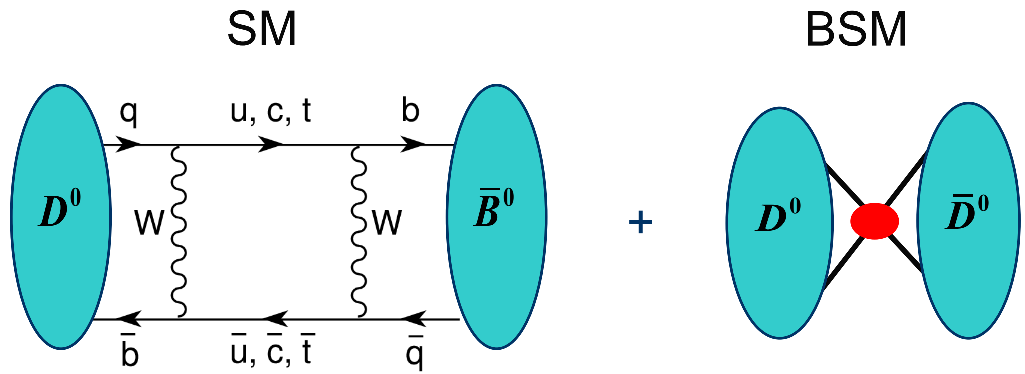


Expect substantial improvements to tree constraints!

2.2 Oscillations of Kaons

- Similar tests with other mesons





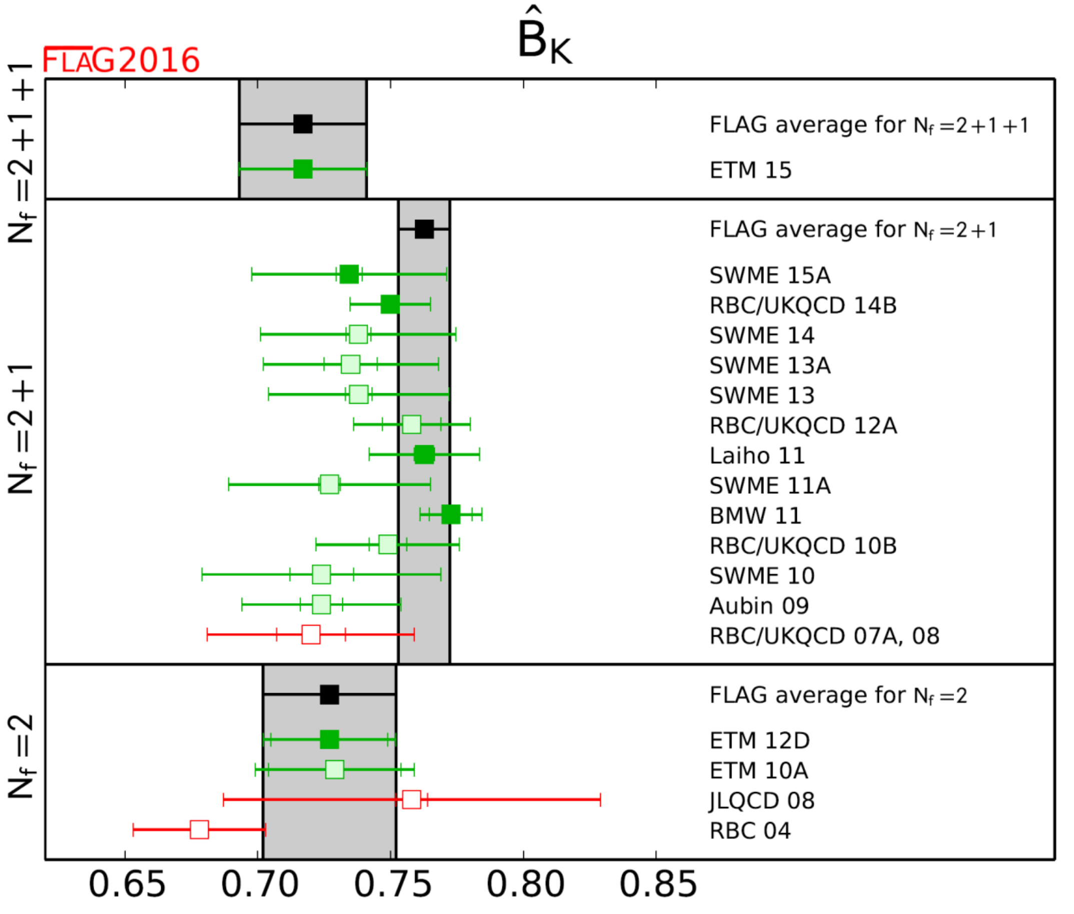
CDF, D0'06, LHCb'11

CP violation in D decays *LHCb'19*

- Stringent constraints on new physics models provided *hadronic* matrix elements known

Lattice results for B_K

$$B_K^{\overline{MS}}(2\,\mathrm{GeV}) = 0.557 \pm 0.007 \quad , \quad \hat{B}_K = 0.763 \pm 0.010 \quad (N_f = 2+1)$$



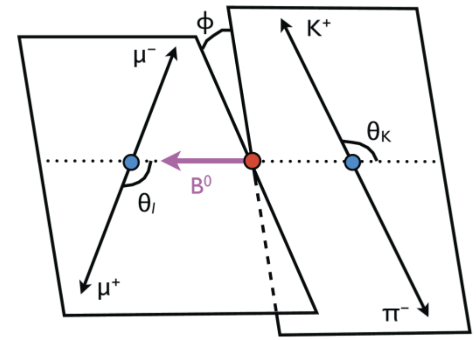
Flavianet Lattice Averaging Group

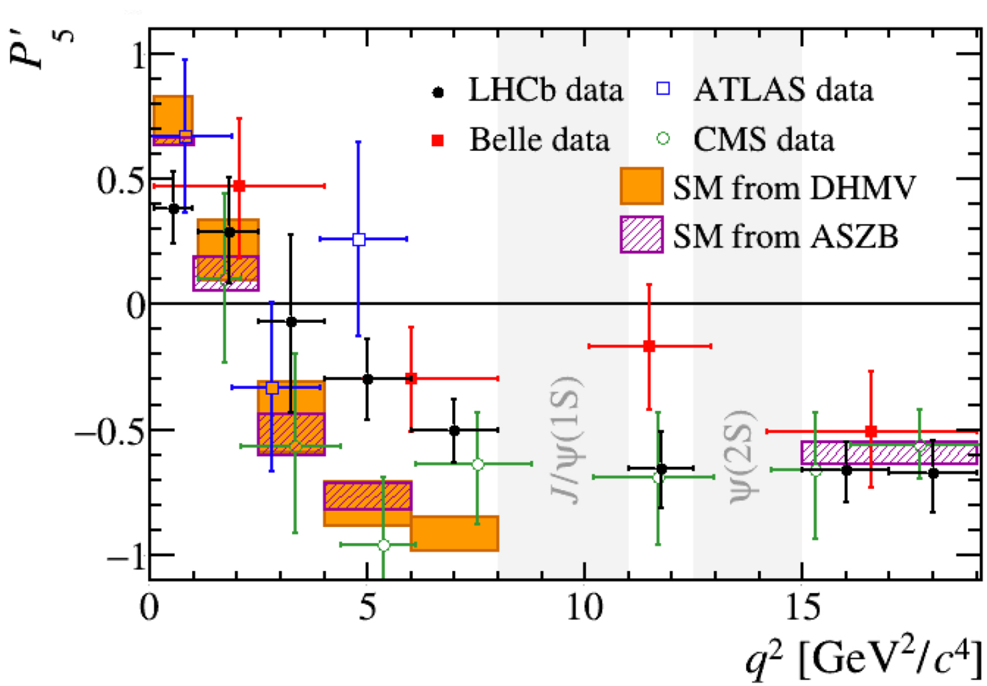
$B \to K^*\mu^+\mu^- \to K\pi\mu^+\mu^-$

$$\frac{1}{d\Gamma/dq^2}\frac{d^4\Gamma}{d\cos\theta_\ell\, d\cos\theta_K\, d\phi\, dq^2} = \frac{9}{32\pi}\Big[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta_K + \frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_\ell$$
$$- F_L\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi$$
$$+ S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + S_5\sin 2\theta_K\sin\theta_\ell\cos\phi$$
$$+ S_6\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi$$
$$+ S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big]$$

$$q^2 = s_{\mu^+\mu^-}$$

$$P'_{i=4,5,6,8} = \frac{S_{j=4,5,7,8}}{\sqrt{F_L(1-F_L)}}$$





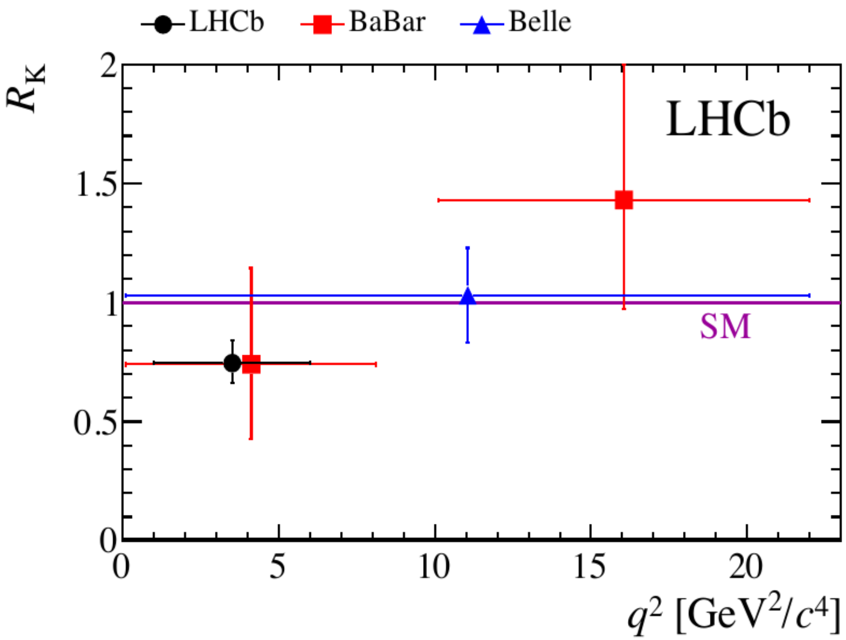
- Build an observable the less sensitive possible to hadronic uncertainties ⇒ P5'
  Only at LO

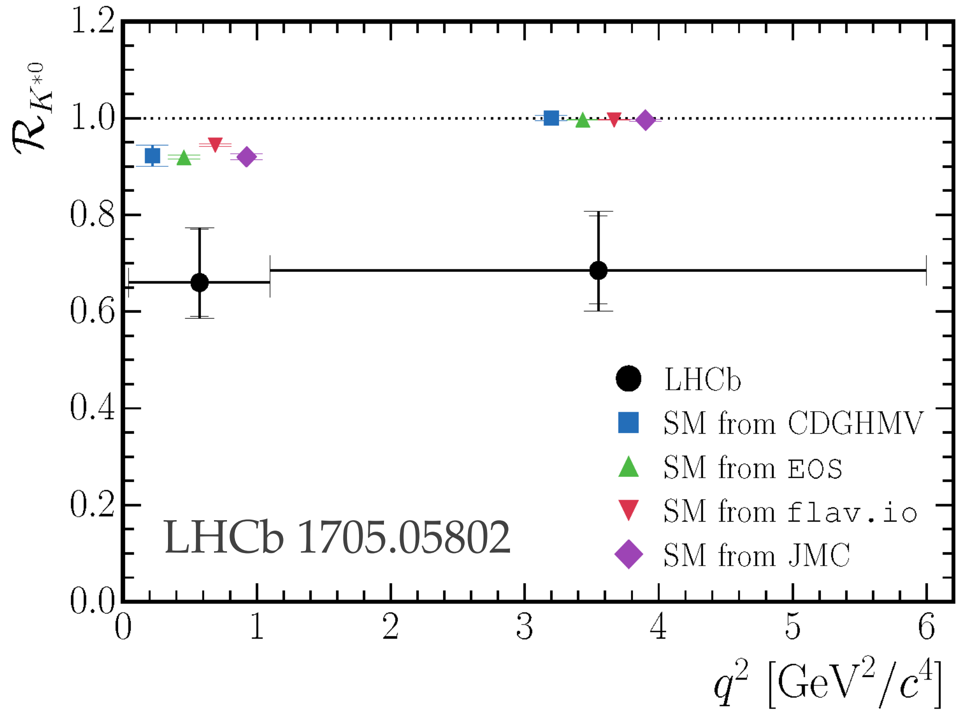
  DHMV: Descotes-Genon et al.'15
  ASZB:

- But new physics contributions involve *hadronic physics*!

R_K, R_{K*}

LHCb 1406.6482



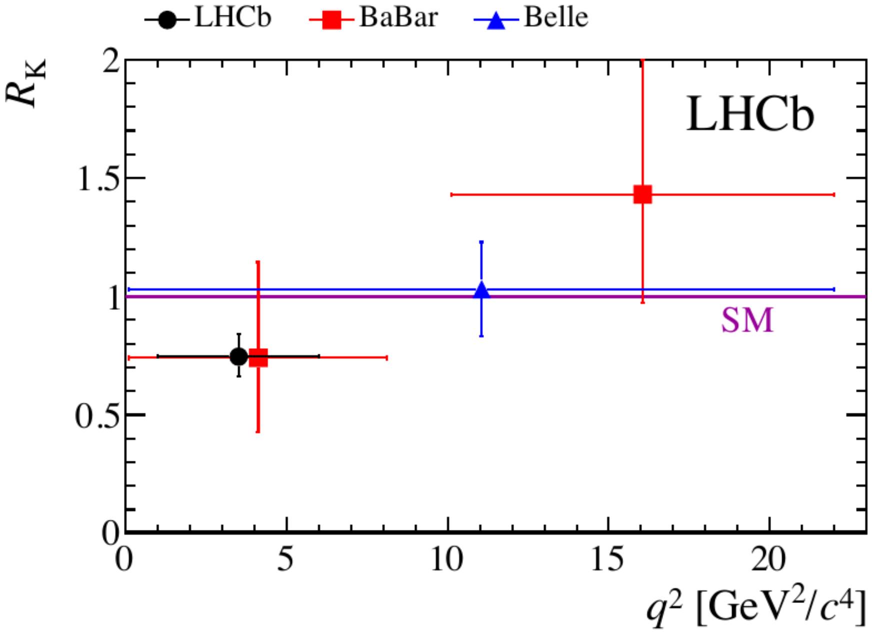


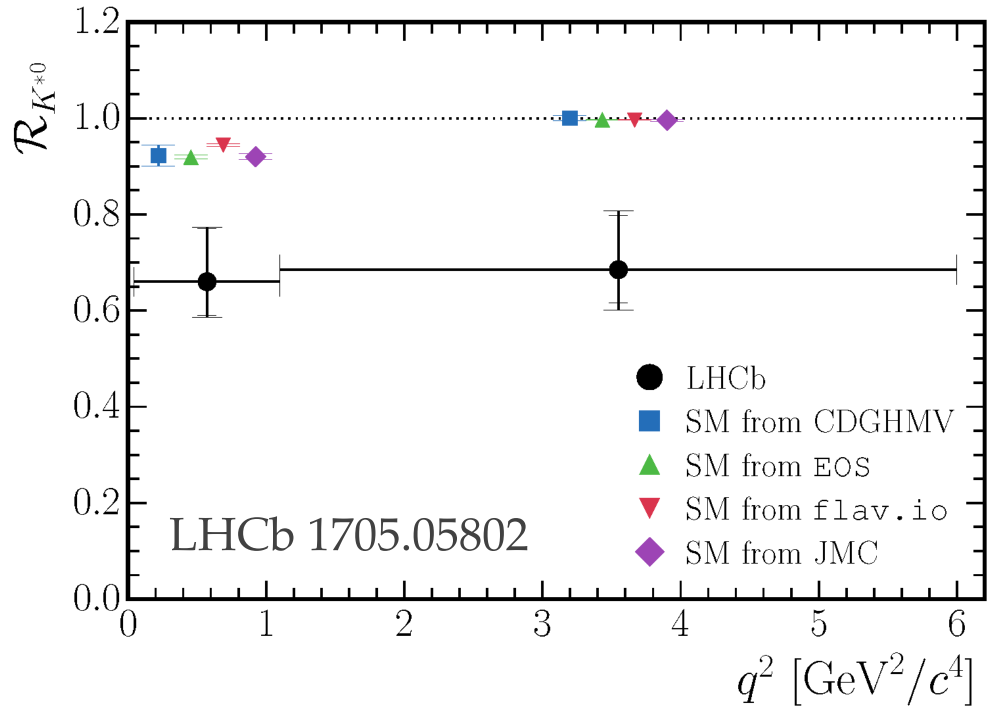
$$R_{K^{(*)}} = \frac{\Gamma(\bar{B} \to \bar{K}^{(*)} \mu^+ \mu^-)}{\Gamma(\bar{B} \to \bar{K}^{(*)} e^+ e^-)}$$

- Hadronic uncertainties cancel in the ratio

R_K, R_{K*}

LHCb 1406.6482





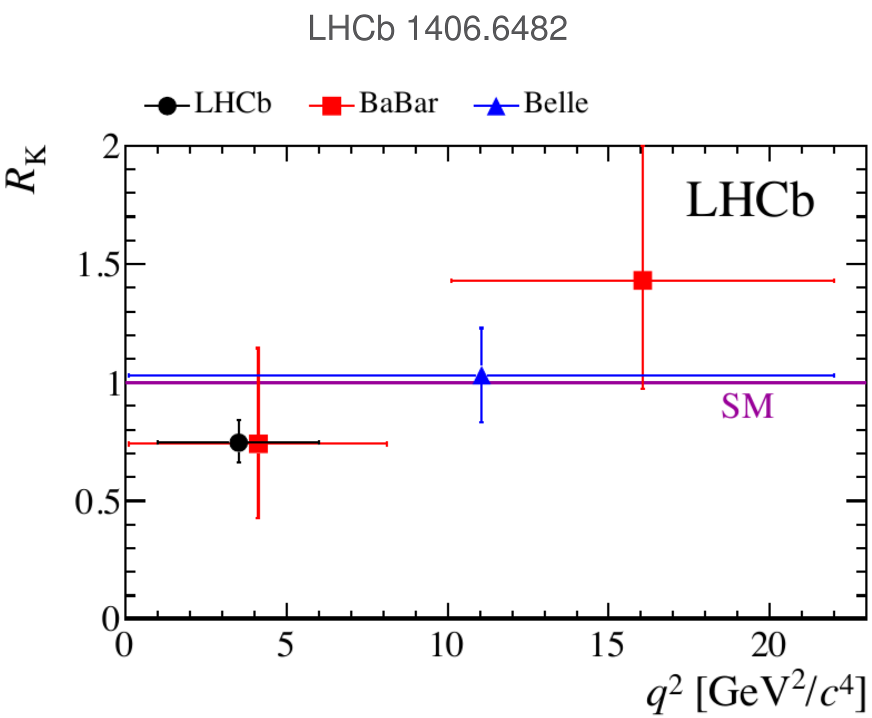
$$R_{K^{(*)}} = \frac{\Gamma(\bar{B} \to \bar{K}^{(*)}\mu^+\mu^-)}{\Gamma(\bar{B} \to \bar{K}^{(*)}e^+e^-)}$$

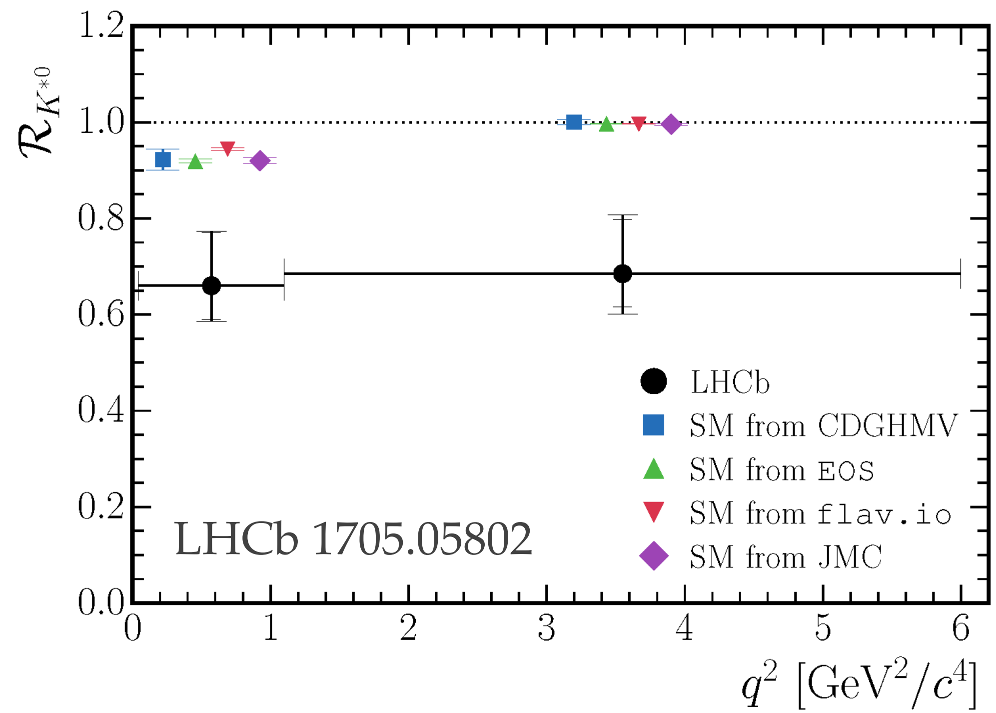
- Hadronic uncertainties cancel in the ratio
- Update from LHCb and Belle

- Original LHC*b* result (2.6σ):

$$R_K = 0.745^{+0.090}_{-0.074}\,(\text{stat}) \pm 0.036\,(\text{syst})$$

R_K, R_{K*}





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- Hadronic uncertainties cancel in the ratio
- Update from LHCb and Belle

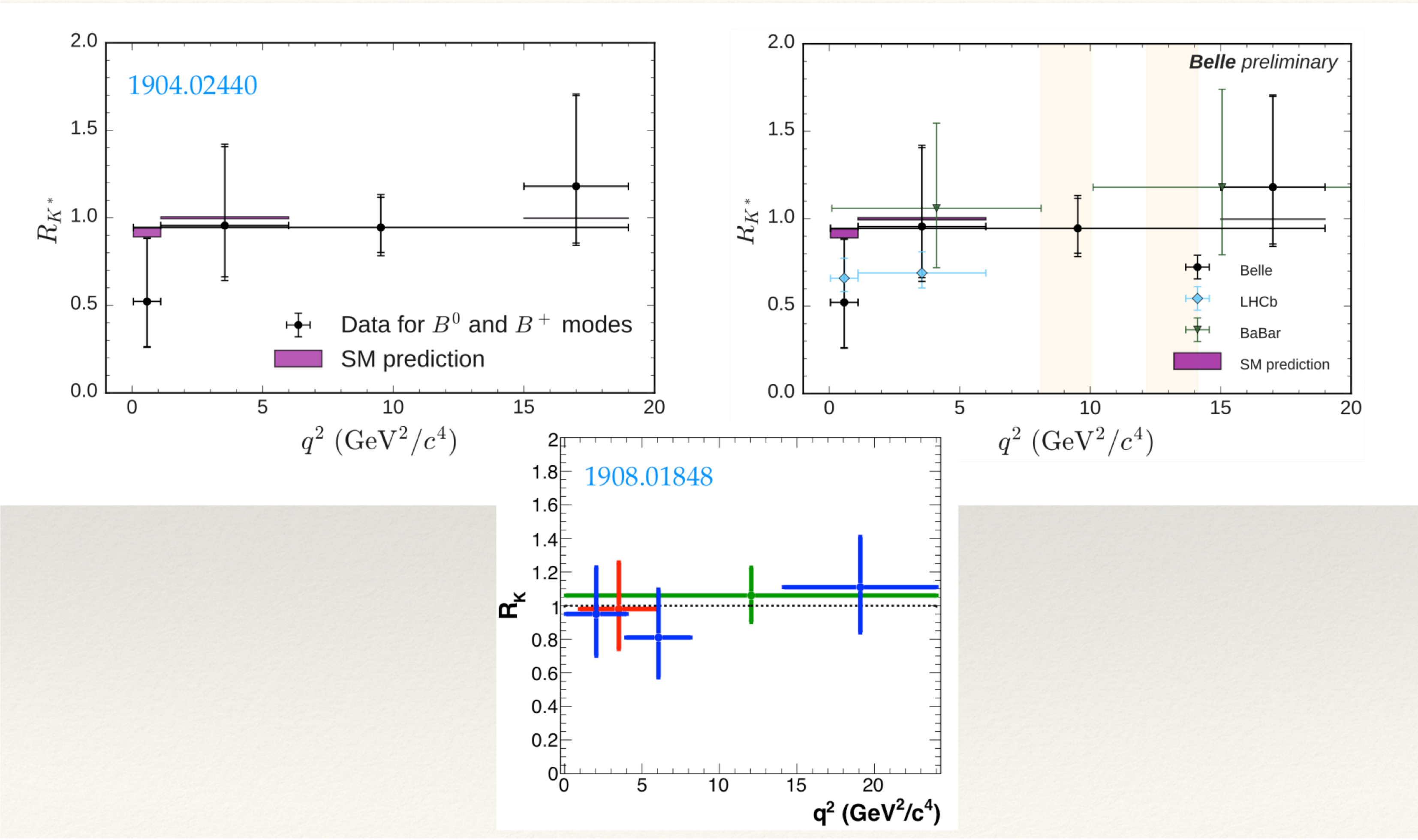
- Original LHC*b* result (2.6σ):

$$R_K = 0.745^{+0.090}_{-0.074}\,(\text{stat}) \pm 0.036\,(\text{syst})$$

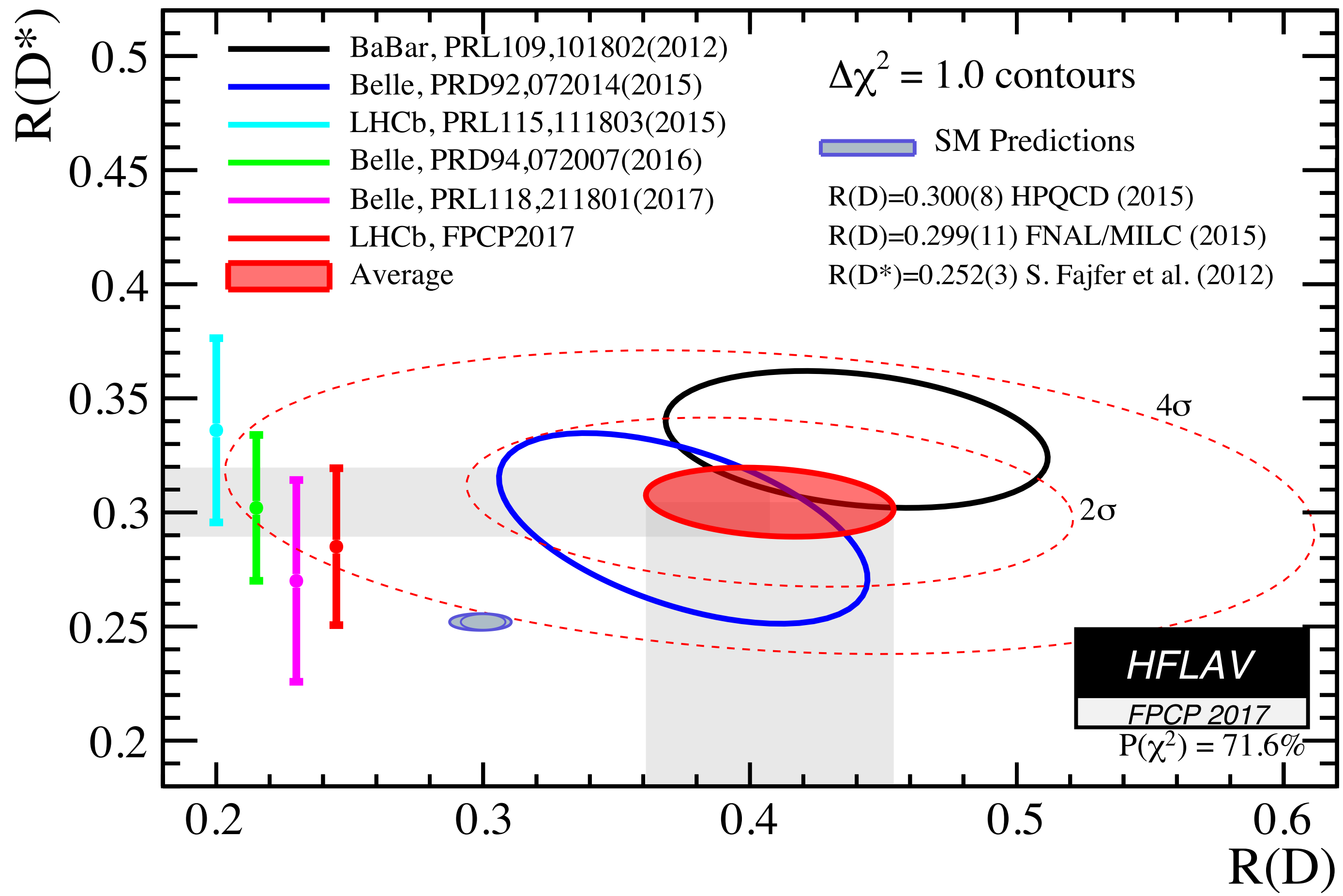
- New result including data until 2016 (2.5σ):

$$R_K = 0.846\,^{+0.060}_{-0.054}\,^{+0.016}_{-0.014}$$

R_K, R_{K*}: Belle results

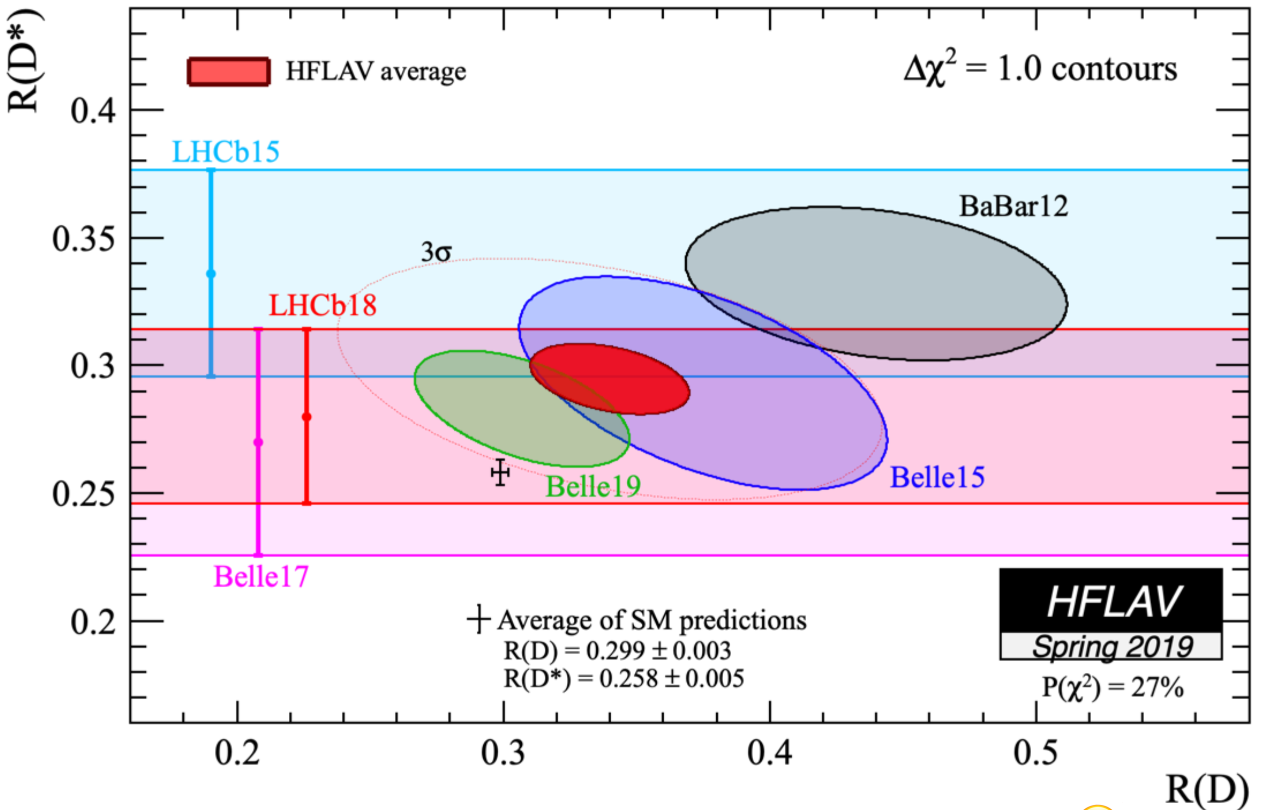


R_D, R_{D*}



$$R_{D^{(*)}} = \frac{\Gamma(\bar{B} \to D^{(*)}\tau\bar{\nu})}{\Gamma(\bar{B} \to D^{(*)}\ell\bar{\nu})}\,; \quad \ell = e, \mu$$

R_D, R_{D*} : recent update from Belle



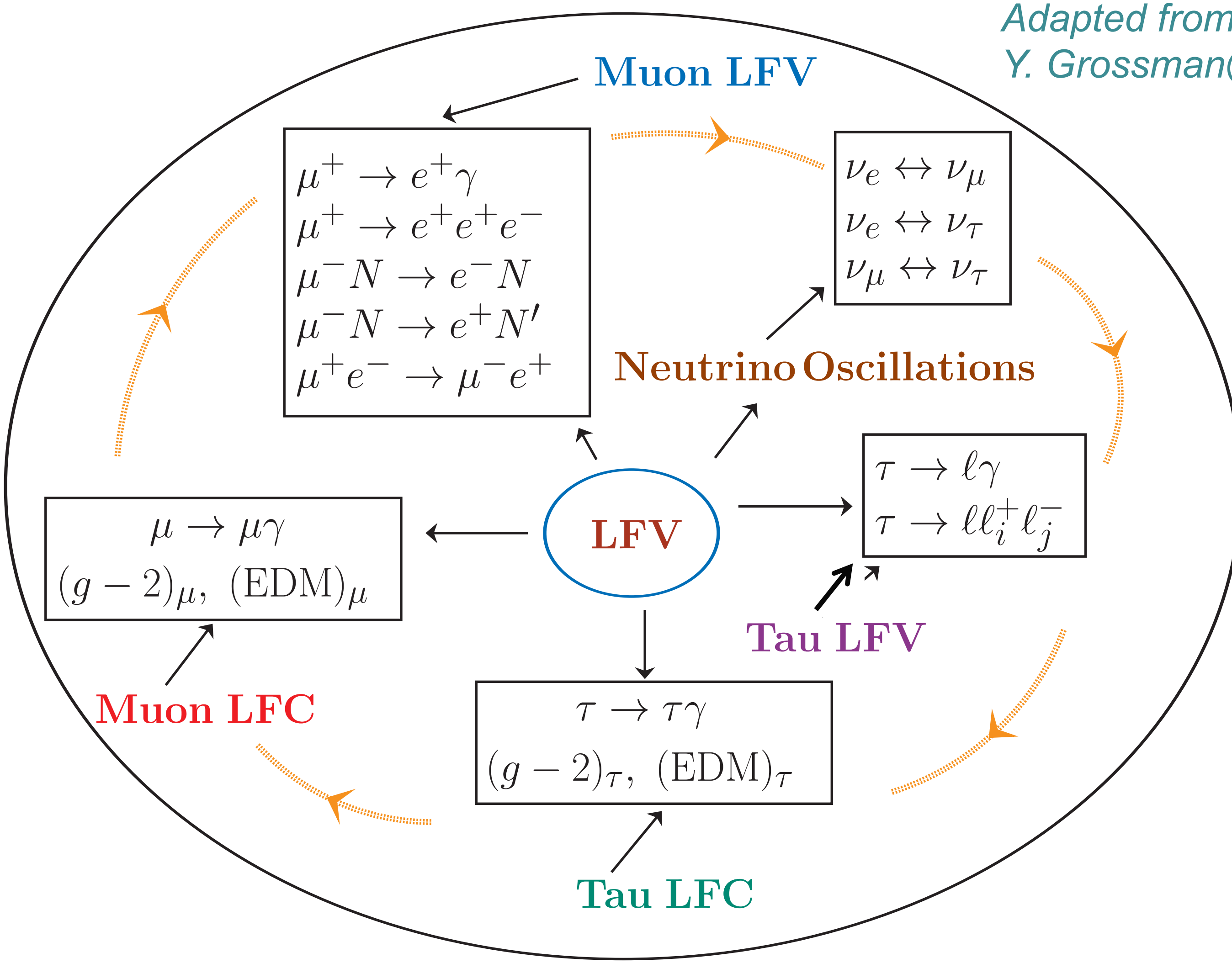
Significance reduced from 4.1 to 3.1σ 🙁

$$\mathcal{R}(D) = 0.307 \pm 0.037 \pm 0.016$$
$$\mathcal{R}(D^*) = 0.283 \pm 0.018 \pm 0.014$$

(Belle 2019: 1.2σ)

Leptons decays

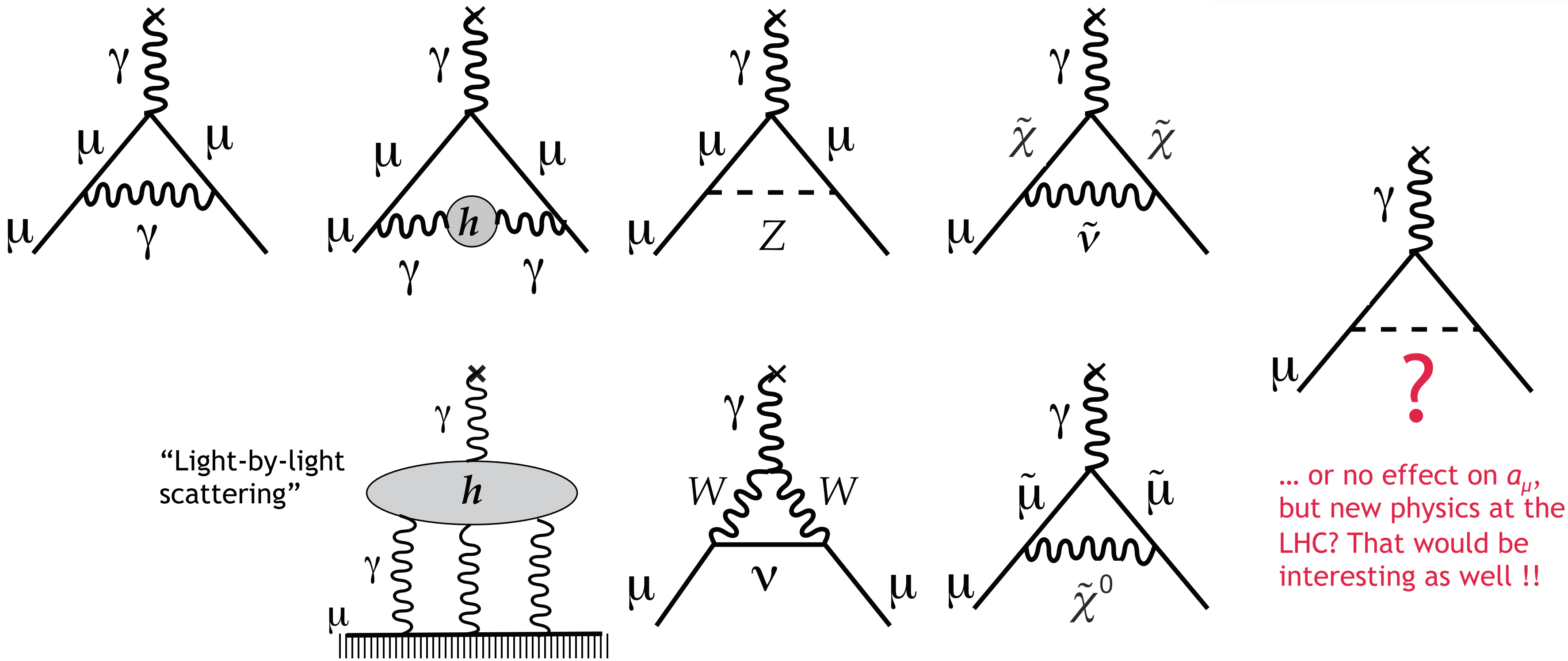
Adapted from Talk by Y. Grossman@CLFV2013



Contribution to $(g-2)_\mu$

Hoecker'11

QED | Hadronic | Weak | SUSY... ? | ... or some unknown type of new physics ?



Need to compute the SM prediction with high precision! ⇨ *Not so easy!*

Hadrons enter virtually through loops!

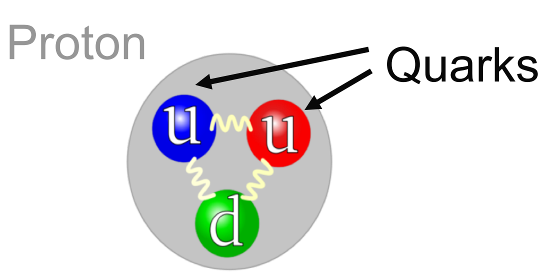
2.1 Quark masses

- Quark masses fundamental parameters of the *QCD Lagrangian*

$$\mathcal{L}_{QCD} = -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \sum_{k=1}^{N_F} \bar{q}_k \left(i\gamma^\mu D_\mu - m_k \right) q_k$$

- No direct experimental access to quark masses due to *confinement*!

- Let us consider the proton: it is not a fundamental particle, but a bound state of 3 quarks



Contrary to naïve expectation, most of its mass comes from *strong force*

Only 1% of its mass comes from the quark masses (Coupling of the quarks to the Higgs boson)

2.1 Quark masses

- Quark masses fundamental parameters of the *QCD Lagrangian*

$$\mathcal{L}_{QCD} = -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \sum_{k=1}^{N_F} \bar{q}_k \left(i\gamma^\mu D_\mu - m_k \right) q_k$$

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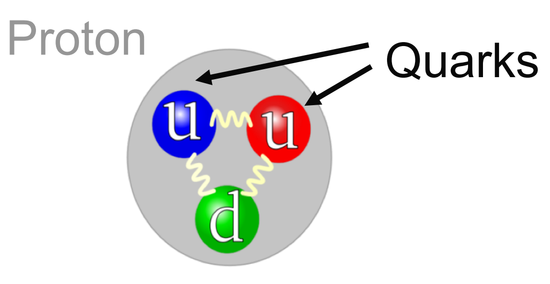


2.6 Why a new dispersive analysis?

- Several new ingredients:
  - New inputs available: extraction $\pi\pi$ phase shifts has improved

    Ananthanarayan et al'01, Colangelo et al'01

    Descotes-Genon et al'01

    Kaminsky et al'01, Garcia-Martin et al'09

  - New experimental programs, precise Dalitz plot measurements

    TAPS/CBall-MAMI (Mainz), WASA-Celsius (Uppsala), WASA-Cosy (Juelich)

    CBall-Brookhaven, CLAS, GlueX (JLab), KLOE I-II (Frascati)

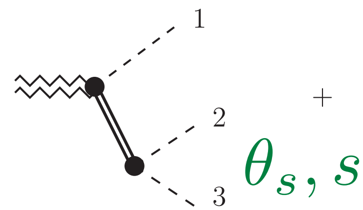
    BES III (Beijing)

  - Many improvements needed in view of very precise data: inclusion of
    - Electromagnetic effects ($\mathcal{O}(e^2 m)$) *Ditsche, Kubis, Meissner'09*
    - Isospin breaking effects

2.7 Method

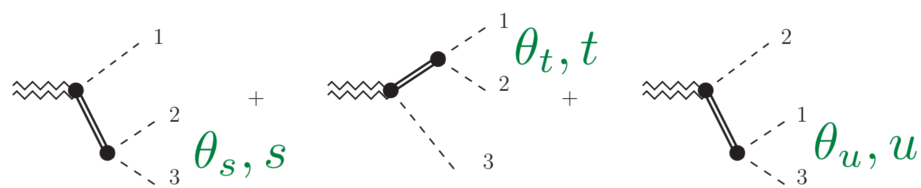
- S-channel partial wave decomposition

$$A_\lambda(s,t) = \sum_J^{\infty} (2J+1) d^J_{\lambda,0}(\theta_s) A_J(s)$$



- One truncates the partial wave expansion : ⇒ Isobar approximation

$$A_\lambda(s,t) = \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_s) f_J(s) + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_t) f_J(t) + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_u) f_J(u)$$



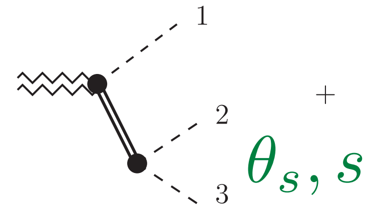
3 BWs (ρ^+, ρ^-, ρ^0) + background term

⇒ Improve to include final states interactions

2.7 Method

- S-channel partial wave decomposition

$$A_\lambda(s,t) = \sum_J^{\infty} (2J+1) d^J_{\lambda,0}(\theta_s) A_J(s)$$



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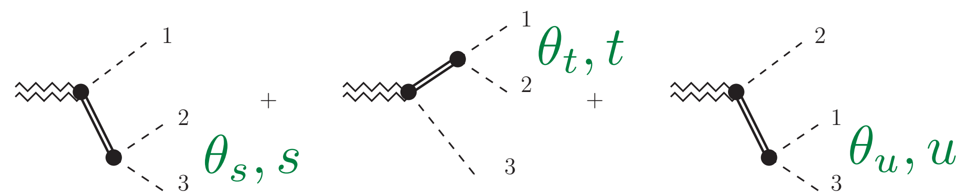
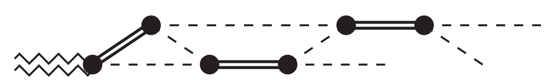




- Use a Khuri-Treiman approach or dispersive approach

⇨ Restore 3 body unitarity and take into account the final state interactions in a systematic way

2.8 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

Fuchs, Sazdjian & Stern'93

Anisovich & Leutwyler'96

  - M_I isospin *I* rescattering in two particles
  - Amplitude in terms of S and P waves ⇨ exact up to NNLO ($\mathcal{O}(p^6)$)
  - Main two body rescattering corrections inside M_I

2.8 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$M(s,t,u) = M_0^0(s) + (s-u)M_1^1(t) + (s-t)M_1^1(u) + M_0^2(t) + M_0^2(u) - \frac{2}{3}M_0^2(s)$$

- Unitarity relation:

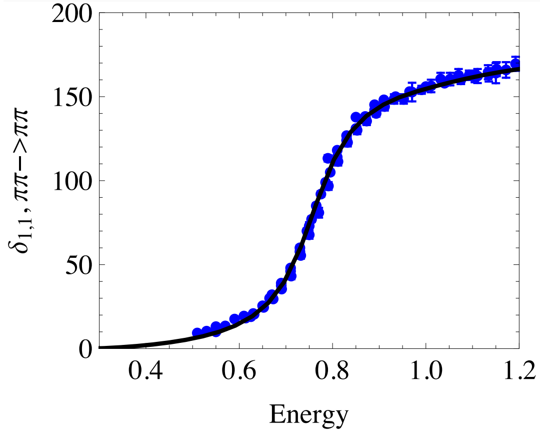
$\pi\pi \to \pi\pi$

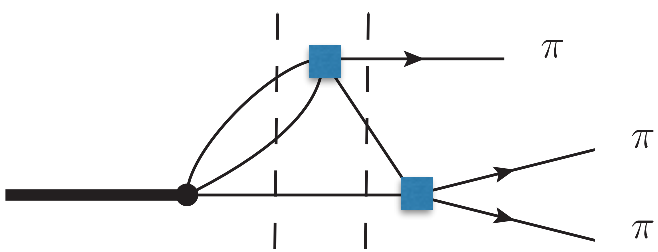
$$disc\left[M_\ell^I(s)\right] = \rho(s)t_\ell^*(s)\left(M_\ell^I(s) + \hat{M}_\ell^I(s)\right)$$

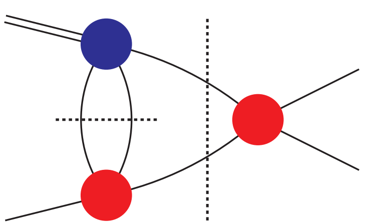
input

right-hand cut   left-hand cut

Roy analysis
Colangelo et al.'01







2.8 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

- Unitarity relation:

$$disc\left[M_\ell^I(s)\right] = \rho(s)t_\ell^*(s)\left(M_\ell^I(s) + \hat{M}_\ell^I(s)\right)$$

- Relation of dispersion to reconstruct the amplitude everywhere:

$$M_I(s) = \Omega_I(s)\left(P_I(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'^n}\frac{\sin\delta_I(s')\,\hat{M}_I(s')}{|\Omega_I(s')|(s'-s-i\varepsilon)}\right)$$

$$\left[\Omega_I(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds'\frac{\delta_I(s')}{s'(s'-s-i\varepsilon)}\right)\right]$$

Omnès function

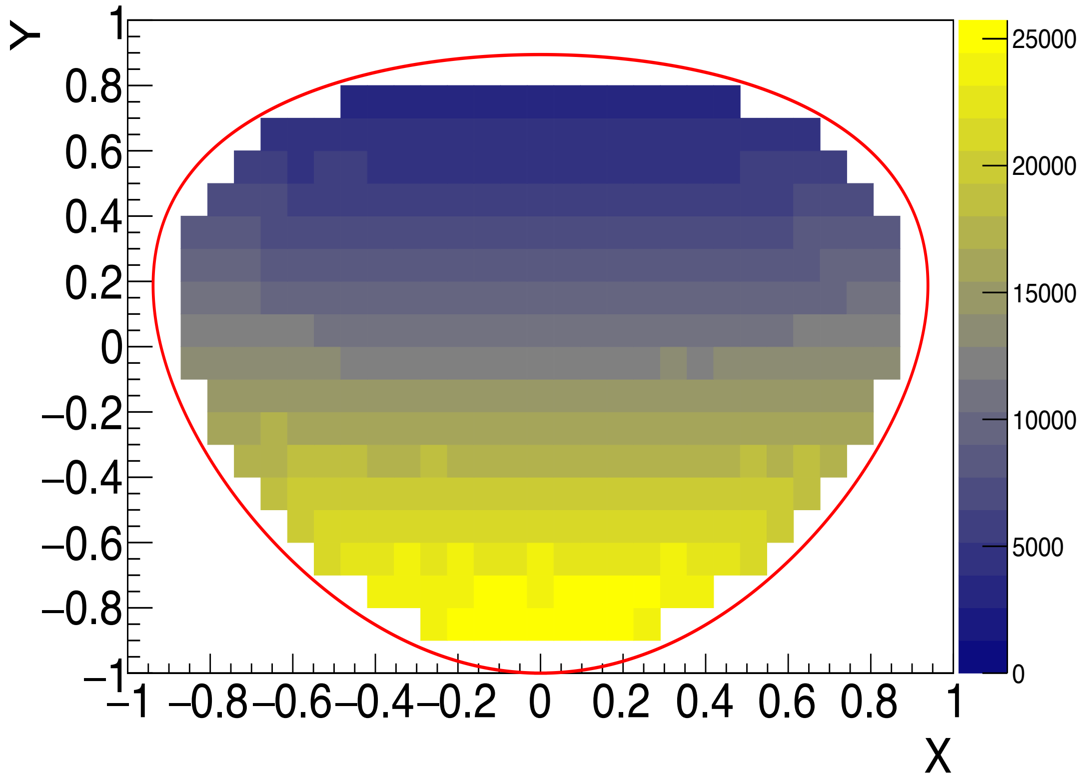
Gasser & Rusetsky'18

- $P_I(s)$ determined from a fit to NLO ChPT + experimental Dalitz plot

2.9 $\eta \to 3\pi$ Dalitz plot

- In the charged channel: experimental data from *WASA, KLOE, BESIII*

KLOE'16



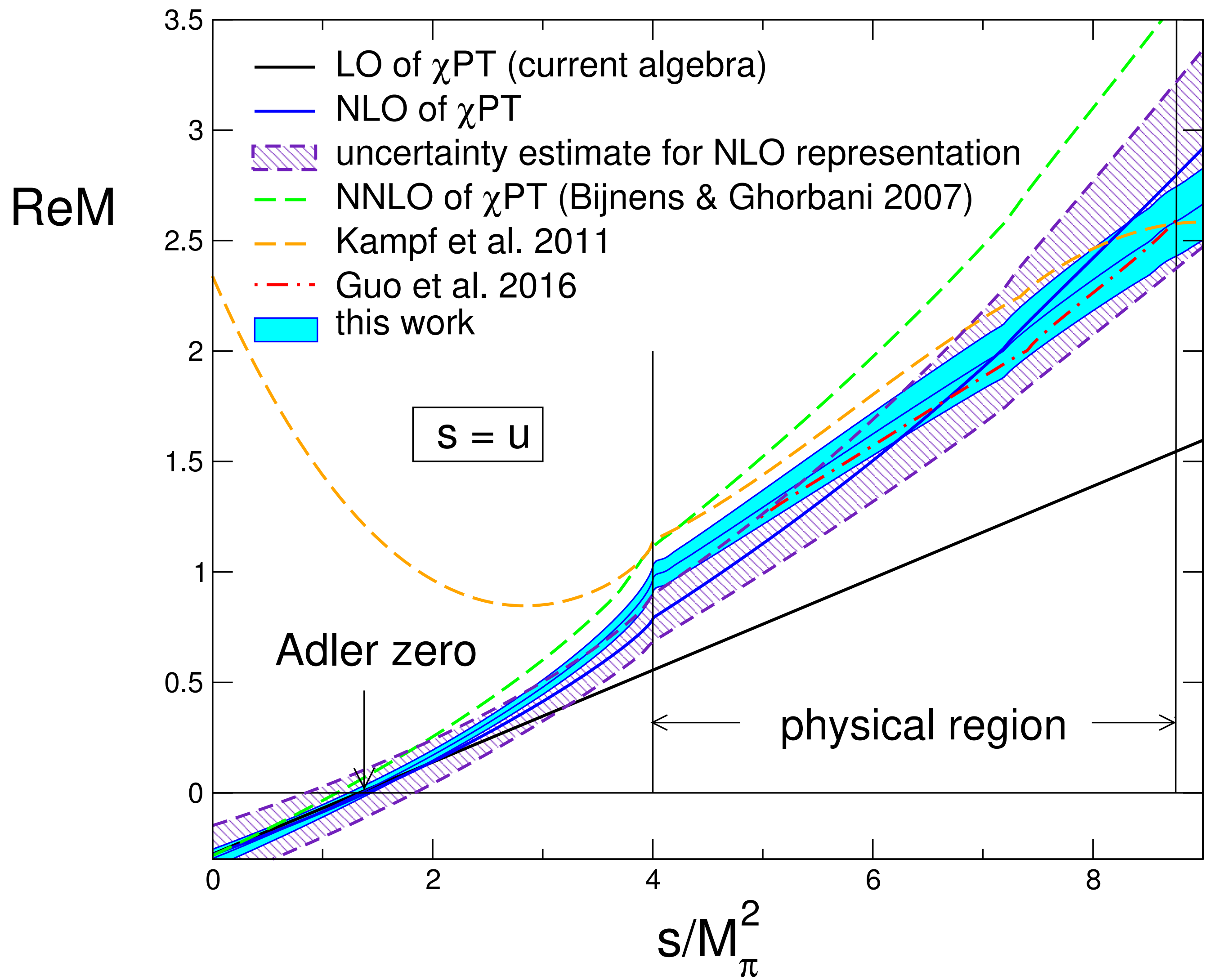
$$X = \sqrt{3}\,\frac{T_+ - T_-}{Q_c} = \frac{\sqrt{3}}{2M_\eta Q_c}(u-t)$$

$$Y = \frac{3T_0}{Q_c} - 1 = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

- New data expected from *CLAS* and *GlueX* with very different systematics

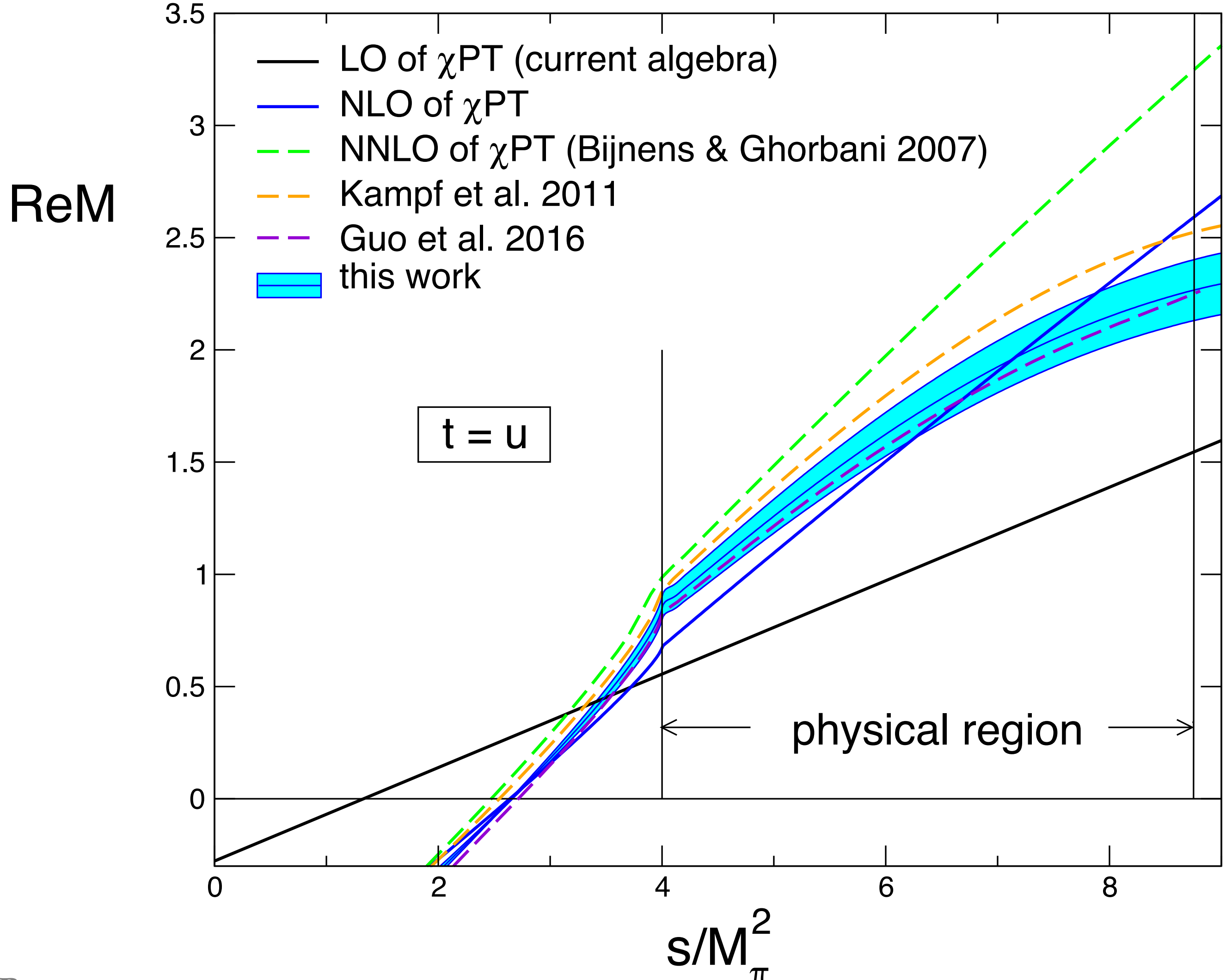
2.10 Results: Amplitude for $\eta \to \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line s = u :



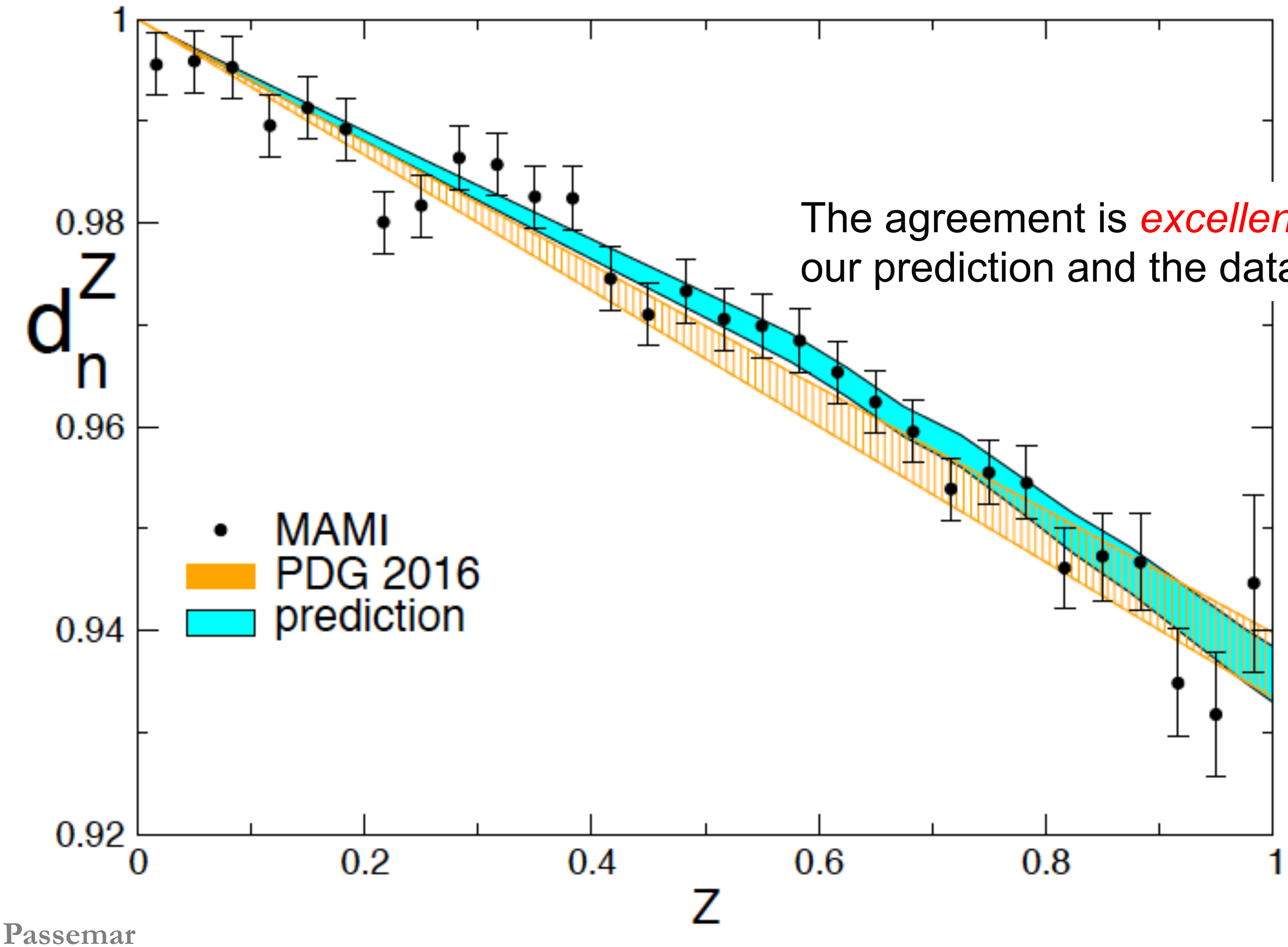
2.10 Results: Amplitude for $\eta \rightarrow \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line t = u :



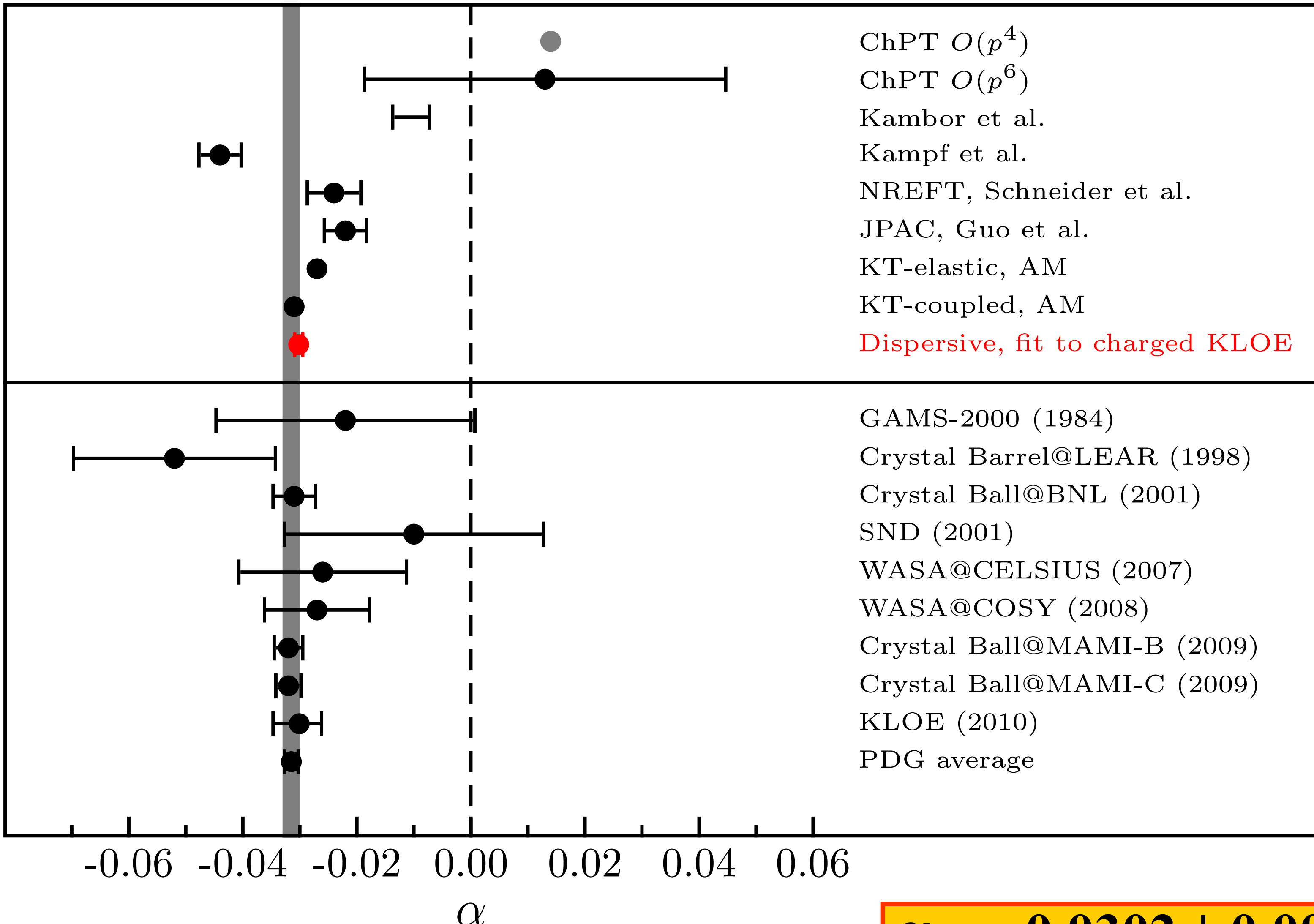
2.11 Z distribution for $\eta \to \pi^0 \pi^0 \pi^0$ decays

- The amplitude squared in the neutral channel is



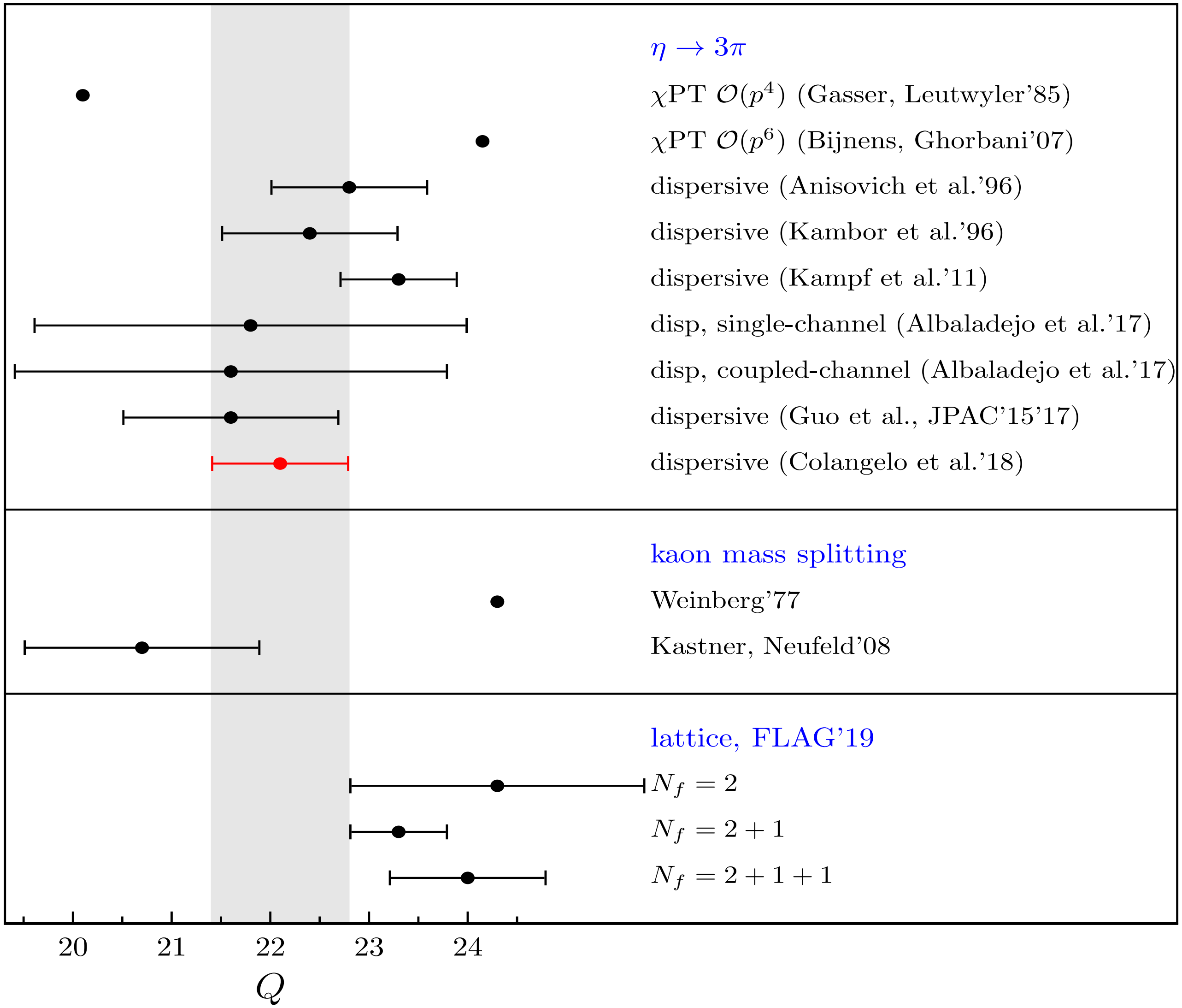
The agreement is *excellent* between our prediction and the data!

2.12 Comparison of results for α



$$\alpha = -0.0302 \pm 0.0011$$

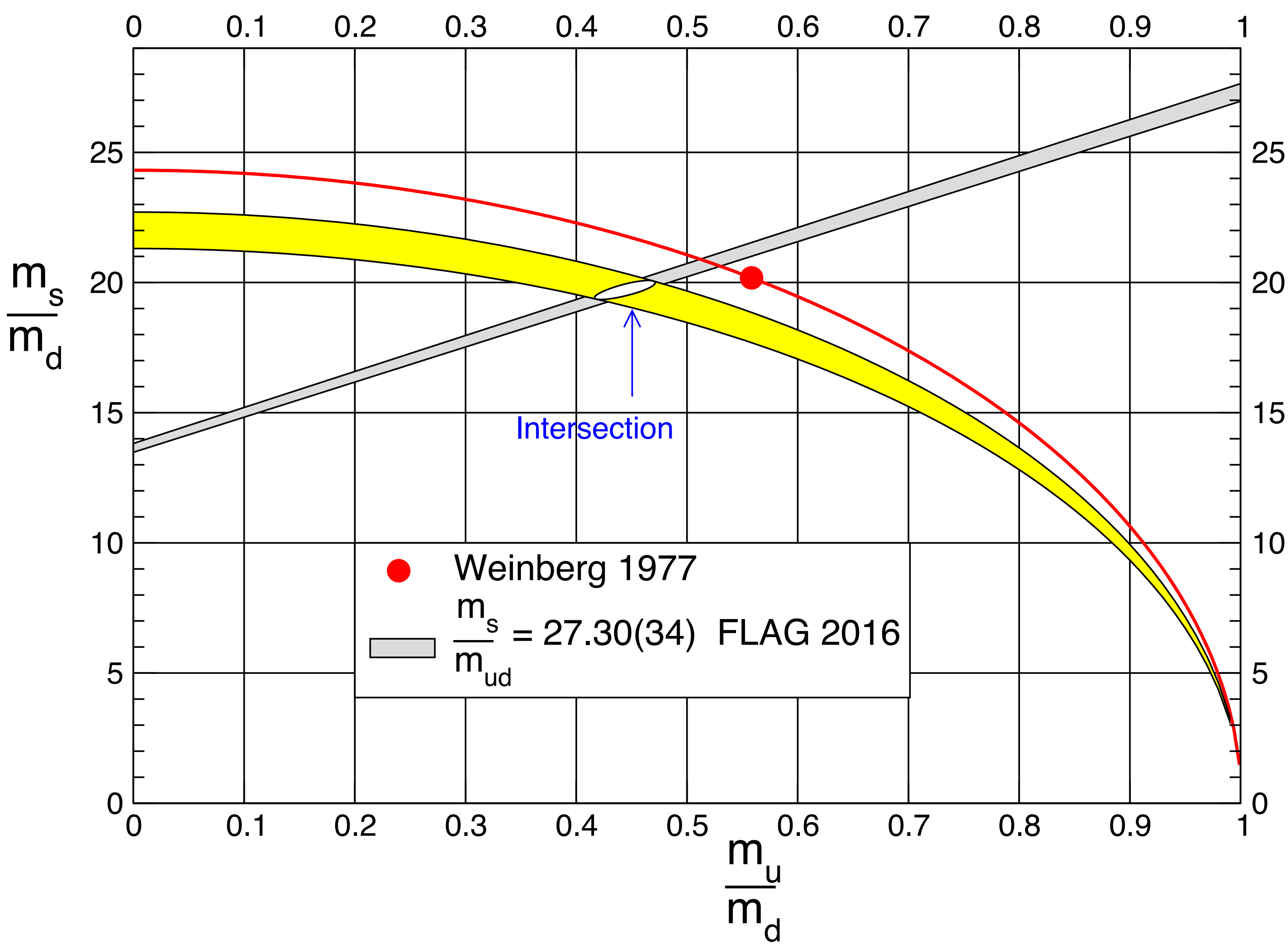
2.13 Quark mass ratio



$Q = 22.1 \pm 0.7$

- No systematics taken into account ⇨ collaboration with experimentalists

2.14 Light quark masses

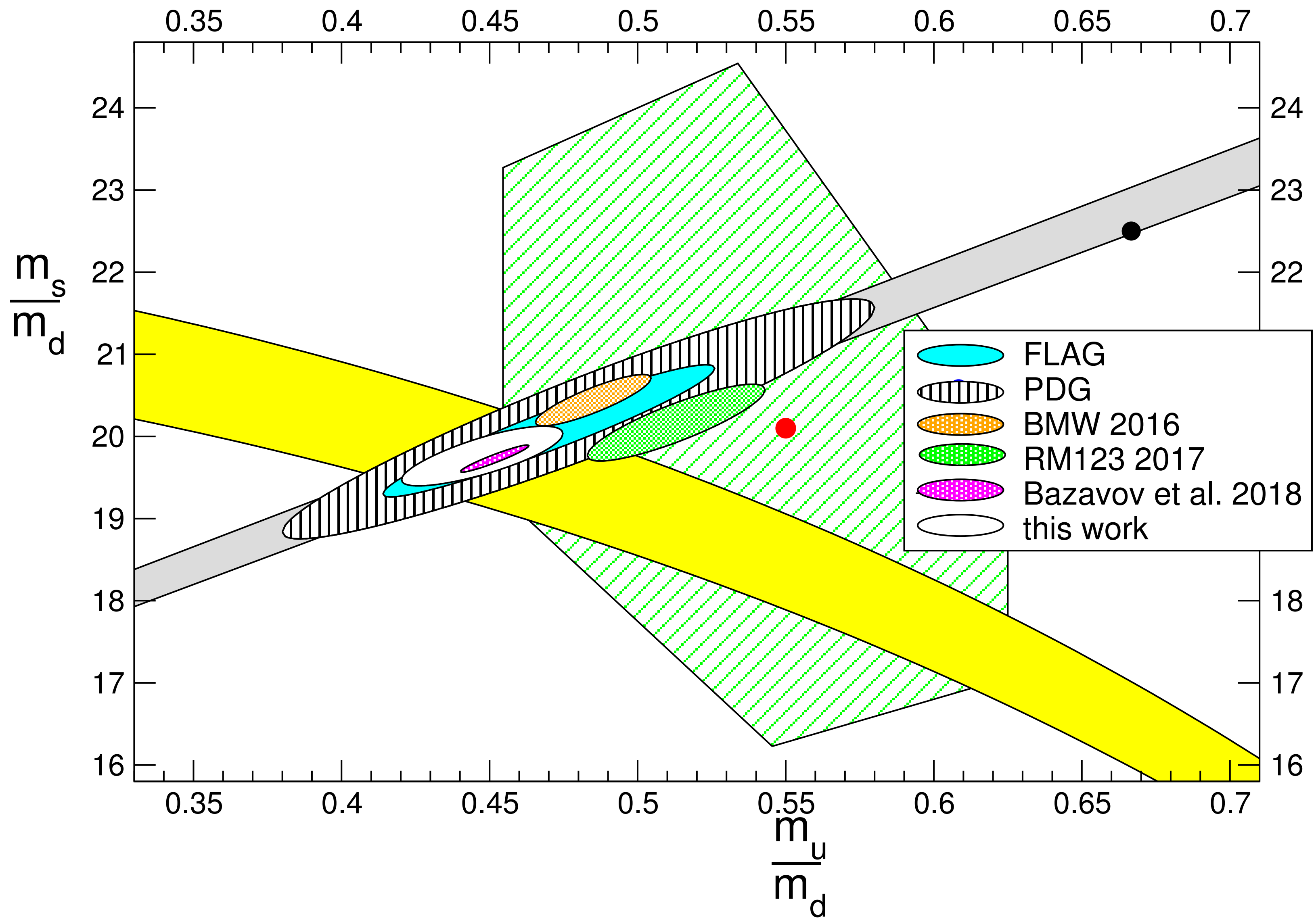


$Q = 22.1 \pm 0.7$

$\frac{m_u}{m_d} = 0.44 \pm 0.03$

- Smaller values for Q ⇨ smaller values for m_s/m_d and m_u/m_d than LO ChPT

2.14 Light quark masses

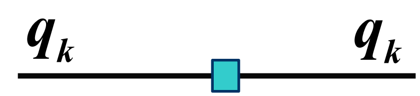


Formulation of QCD

Dynamics: The Lagrangien

- Build all the invariants under *SU(3)$_C$* with the quarks

$$\Rightarrow \quad \mathcal{L}_0 = \sum_{k=1}^{N_F} \bar{q}_k \left(i\gamma^\mu \partial_\mu - m_k \right) q_k$$

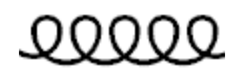


invariant under global SU(3)$_C$: $q_k^\alpha \rightarrow \left(q_k^\alpha\right)' = U^\alpha{}_\beta \, q_k^\beta$

with $U = \exp\left(-ig_S \frac{\lambda_a}{2}\theta_a\right)$ and λ_a the generators of SU(3)$_C$: $\left[\lambda^a, \lambda^b\right] = 2if^{abc}\lambda^c$

- Gauge the theory: SU(3)$_C$ → local ⇒ $\theta_a \rightarrow \theta_a(x)$

⇒ 8 different independent gauge fields: G_μ^a the *gluons*

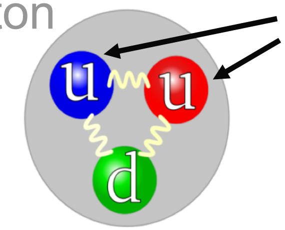


$$\partial_\mu q_k \rightarrow D_\mu q_k \equiv \Big[\partial_\mu - ig_s \underbrace{\frac{\lambda_a}{2} G_\mu^a(x)}_{G_\mu(x)}\Big] q_k$$

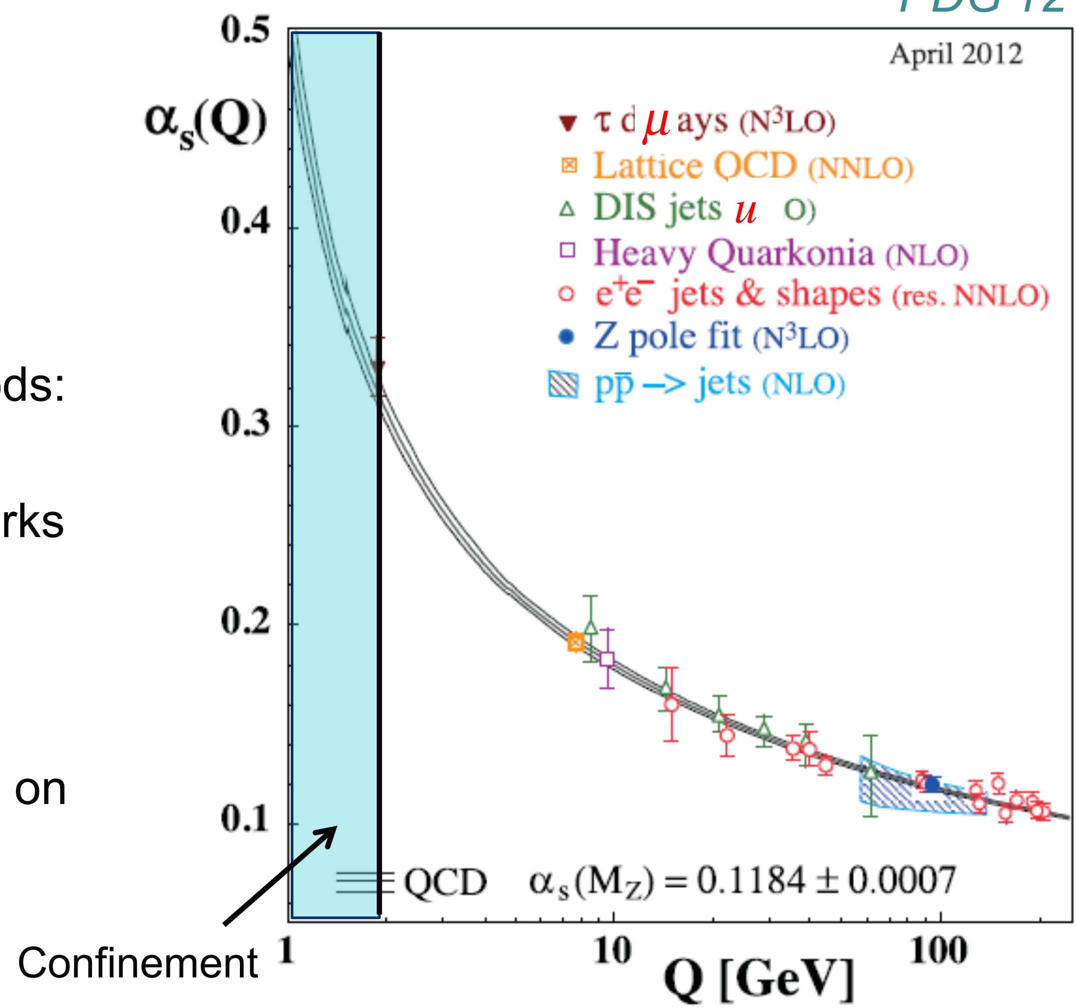
1.4 Strong interaction

- Looking for new physics in hadronic processes ⇨ not direct access to quarks due to confinement

Proton



PDG'12



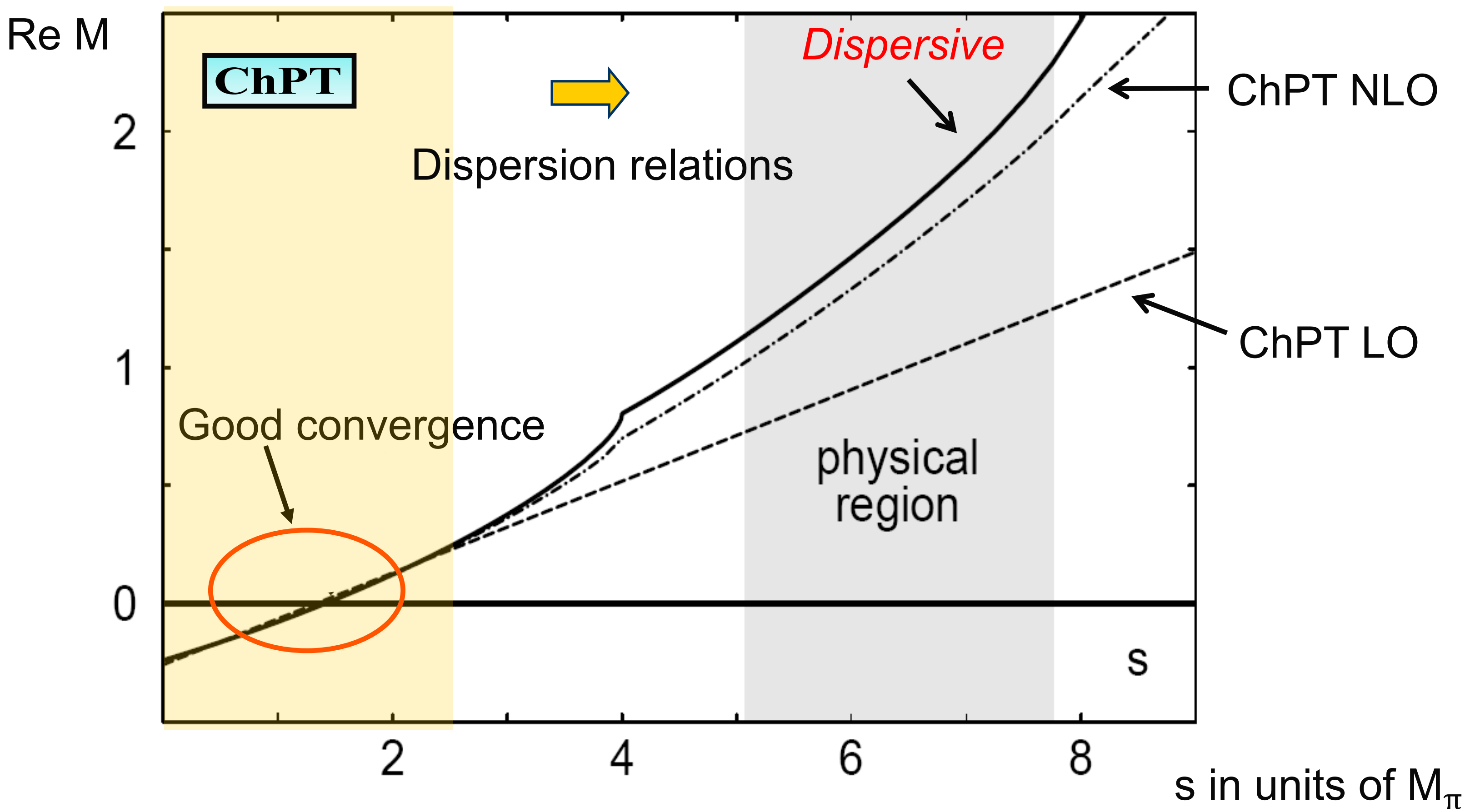
➢ Model independent methods:
  - Effective field theory
    Ex: ChPT for light quarks
  - Dispersion relations
  - Numerical simulations on the lattice

⇨ *Hadronic Physics*

Dispersive approach

- Dispersion Relations: extrapolate ChPT at higher energies

Anisovich & Leutwyler'96

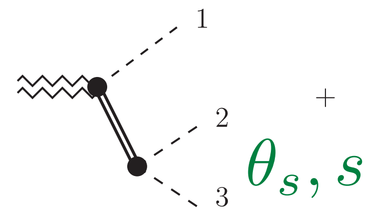


- Important corrections in the physical region taken care of by the *dispersive treatment!*

Method

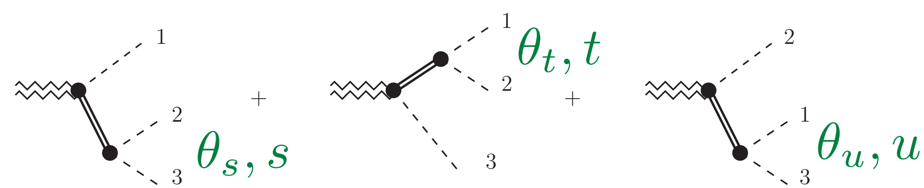
- S-channel partial wave decomposition

$$A_\lambda(s,t) = \sum_J^{\infty} (2J+1) d^J_{\lambda,0}(\theta_s) A_J(s)$$



- One truncates the partial wave expansion : ➡ Isobar approximation

$$A_\lambda(s,t) = \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_s) f_J(s) + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_t) f_J(t) + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_u) f_J(u)$$



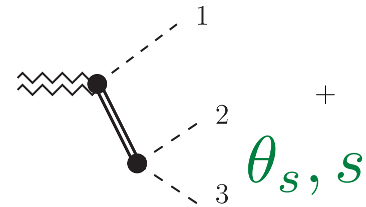
3 BWs (ρ^+, ρ^-, ρ^0) + background term

➡ Improve to include final states interactions

Method

- S-channel partial wave decomposition

$$A_\lambda(s,t) = \sum_J^{\infty} (2J+1) d^J_{\lambda,0}(\theta_s) A_J(s)$$



- One truncates the partial wave expansion : ➡ Isobar approximation

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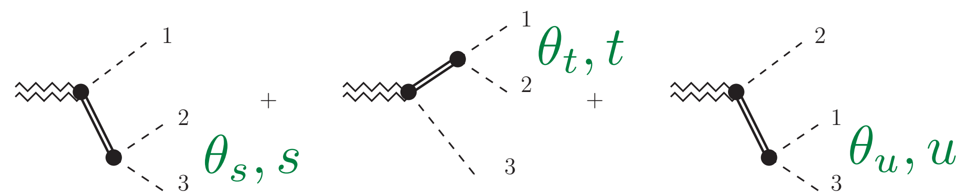

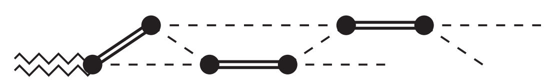

- Use a Khuri-Treiman approach or dispersive approach

➡ Restore 3 body unitarity and take into account the final state interactions in a systematic way

Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

Fuchs, Sazdjian & Stern'93
Anisovich & Leutwyler'96

- M_I isospin *I* rescattering in two particles
- Amplitude in terms of S and P waves ⇨ exact up to NNLO ($\mathcal{O}(p^6)$)
- Main two body rescattering corrections inside M_I

Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

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$$\left[\Omega_I(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}ds'\frac{\delta_I(s')}{s'(s'-s-i\varepsilon)}\right)\right]$$

Omnès function

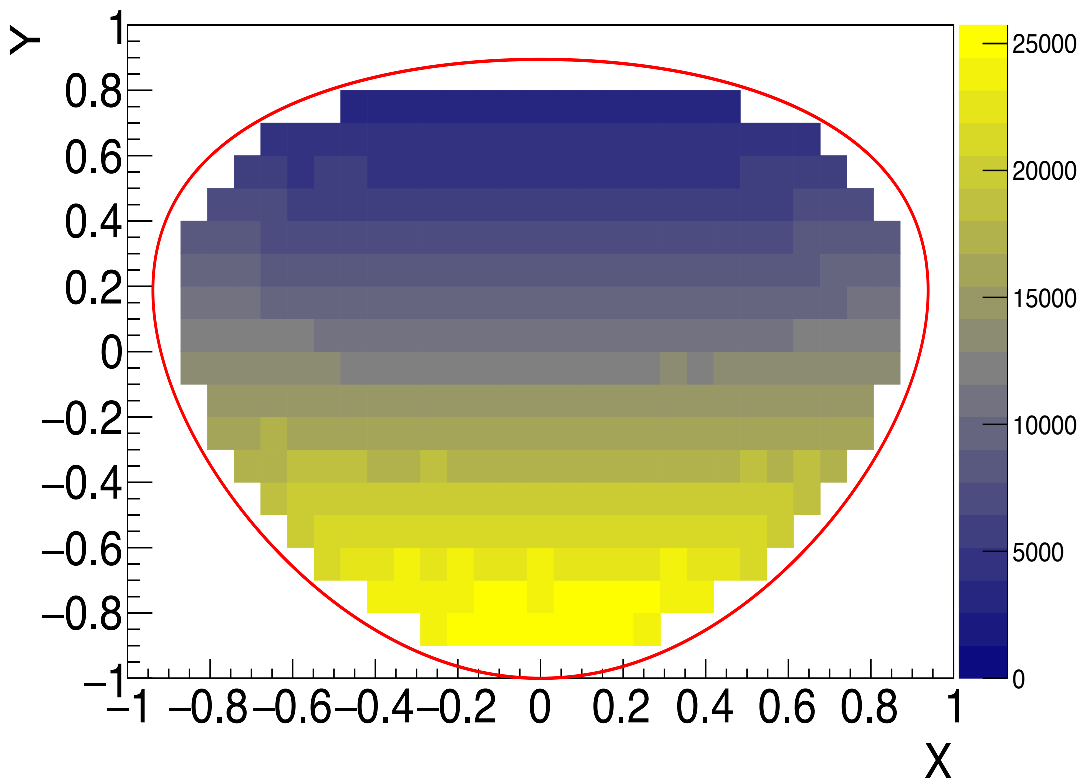
Gasser & Rusetsky'18

- $P_I(s)$ determined from a fit to NLO ChPT + experimental Dalitz plot

$\eta \to 3\pi$ Dalitz plot

- In the charged channel: experimental data from *WASA, KLOE, BESIII*

KLOE'16



$$X = \sqrt{3}\,\frac{T_+ - T_-}{Q_c} = \frac{\sqrt{3}}{2M_\eta Q_c}\left(u - t\right)$$

$$Y = \frac{3T_0}{Q_c} - 1 = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

- New data expected from *CLAS* and *GlueX* with very different systematics

Which value of Q^2 impact neutrino data?

- The experimental results point towards a larger value of the axial form factor $M_A \sim 1.35$ GeV
- If true, the value of M_A saturates the cross section leaving little room for multi nucleon effects
- Is the dipole physically motivated?

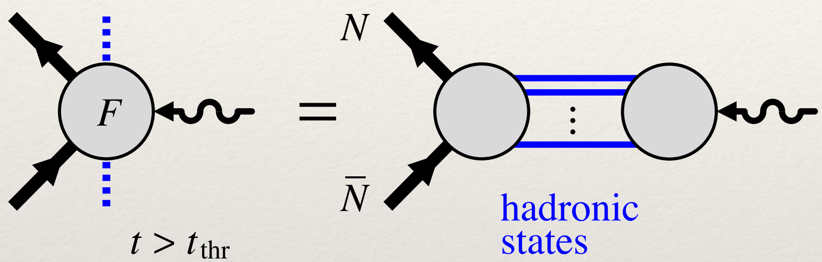
$$F_A(q^2) = \frac{F_A(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2}$$

The parametrisation has an impact on different q^2 dependence ranges on the neutrino data

Improving the Form Factor parametrization

- For intermediate energy region: Can try to use ***VMD***

  - ***Analytical structure*** of FF (e.g. F_1 or F_A)

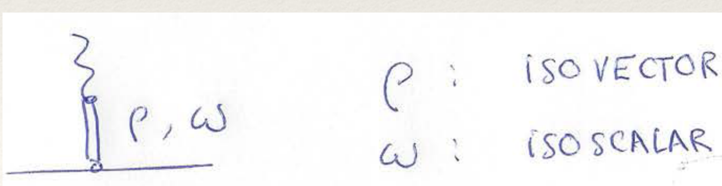


Photon or W sees proton through all hadronic states (with vector or axial-vector Quantum Number)

Isovector: $\pi\pi$ (incl. ρ), 4π, $K\bar{K}$, ...
Isoscalar: 3π (incl. ω), $K\bar{K}$ (incl. ϕ), ...

Processes in unphysical region $t < 4\, m_N^2$

  - Resonances (Vector Mesons)



For F_A (Axial Vector Mesons)

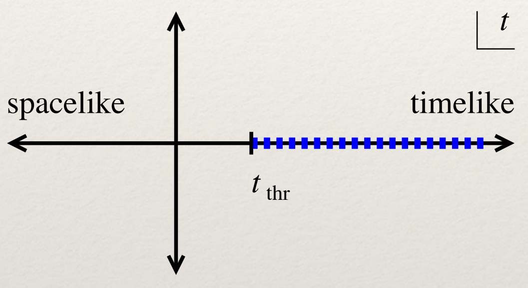
$a_1(1230)$ and $a_1'(1647)$

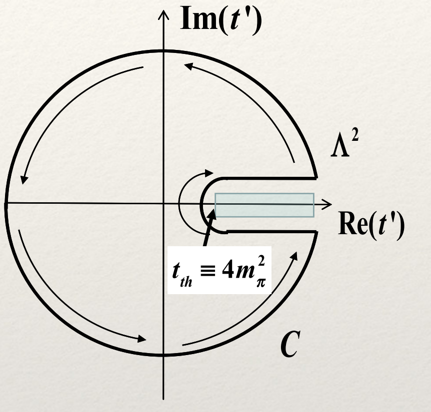
Masjuan et al.'12

$$F_A(t) = g_A \frac{m_{a_1}^2\, m_{a_1'}^2}{(m_{a_1}^2 - t)(m_{a_1'}^2 - t)}$$

Improving the Form Factor parametrization

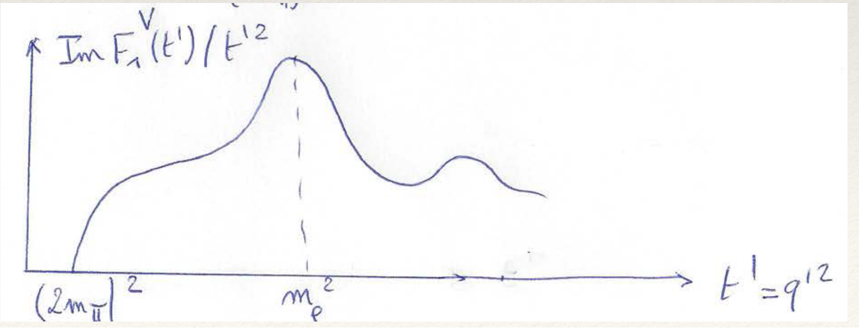
- For intermediate energy region: Can try to use ***VMD***, e.g. EM FF
  - ***Dispersion Relations***





  - Use spectral function from theory or from experiment

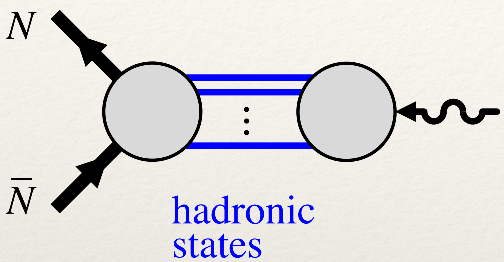
Frazer &Fulco'60, Hohler et al'75



$$F_i(t) = \int_{t_{\rm thr}}^{\infty} \frac{dt'}{\pi} \frac{{\rm Im}\, F_i(t')}{t' - t - i0}$$

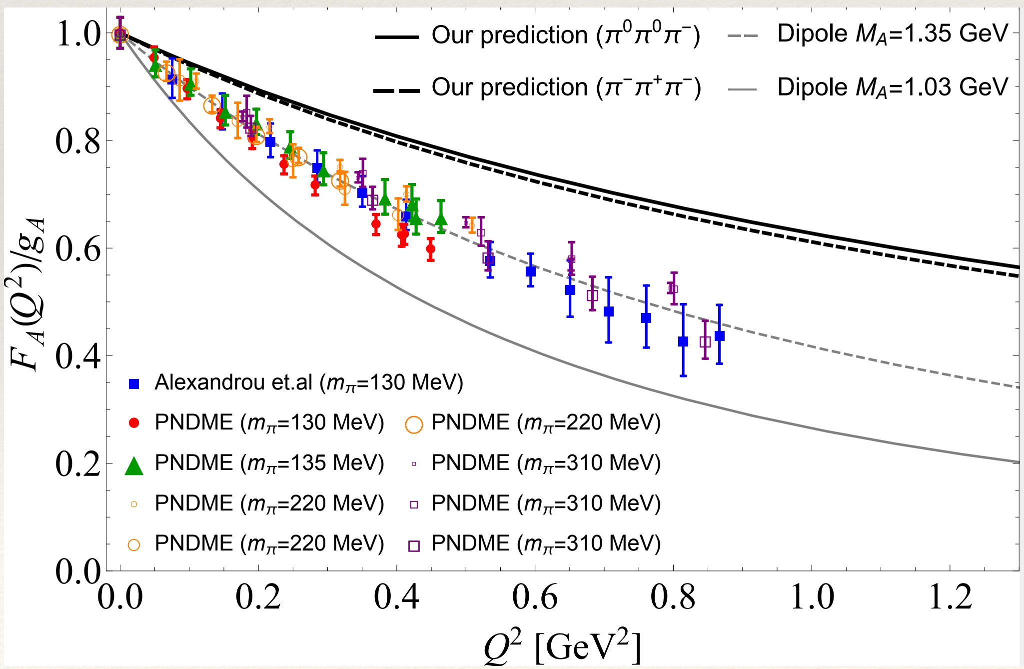
Improving the Form Factor parametrization

- How to connect to the nucleon?



- Take a constant g_A

$$F_A(q^2) = g_A \cdot f_{A\to 3\pi}\left(q^2\right)$$



Does not work!